Initial Model for the Impact of Social Distancing on CoVID-19 Spread

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Abstract

The initial stages of the CoVID-19 coronavirus pandemic all around the world exhibit a nearly exponential rise in the number of infections with time. Planners, governments, and agencies are scrambling to figure out "*How much? How bad?*" and how to effectively treat the potentially large numbers of simultaneously sick people. Modeling the CoVID-19 pandemic using an exponential rise implicitly assumes a nearly unlimited population of uninfected persons (*"dilute pandemic"*). Once a significant fraction of the population is infected (*"saturated pandemic"*), an exponential growth no longer applies. A new model is developed here, which modifies the standard exponential growth function to account for factors such as *Social Distancing*. A *Social Mitigation Parameter [SMP]* α_S is introduced to account for these types of society-wide changes, which can modify the standard exponential growth function, as follows:

$$N(t) = N_o \exp[+K_o\, t\, /\, (1 + \alpha_S\, t\,)]\,.$$

The *doubling-time* $t_{dbl} = (\ln 2)/K_o$ can also be used to substitute for K_o, giving a $\{t_{dbl},\ \alpha_S\}$ parameter pair for comparing to actual CoVID-19 data. This model shows how the number of CoVID-19 infections can self-limit before reaching a *saturated pandemic* level. It also provides estimates for: (a) the timing of the *pandemic peak*, (b) the maximum number of new daily cases that would be expected, and (c) the expected total number of CoVID-19 cases. This model shows fairly good agreement with the presently available CoVID-19 pandemic data for several individual States, and the for the USA as a whole (*6 Figures*), as well as for various countries around the World (*9 Figures*). An augmented model with two *Mitigation Parameters*, α_S and β_S, is also developed, which can give better agreement with the daily new CoVID-19 data. Data-to-model comparisons also indicate that using α_S by itself likely provides a worst-case estimate, while using both α_S and β_S likely provides a best-case estimate for the CoVID-19 spread.

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1 Introduction

The Coronavirus 2019 disease (CoVID-19), caused by the SARS-CoV-2 (Severe Acute Respiratory Syndrome Coronavirus 2) pathogen, is now a world-wide pandemic. In many localities, the number of cases $N(t)$ was found to have an initial period of nearly exponential growth:

$$N(t) = N_o \exp[+K_D\, t]\,, \qquad\qquad\qquad [1.1a]$$
$$K_D \equiv (\ln 2)\,/\,t_D\,, \qquad\qquad\qquad [1.1b]$$

aside of the first few cases, which may be untraceable. In Eqs. [1.1a]-[1.1b], N_o is the initial number of infections at the $t = 0$ start of data tracking, K_D is an exponential growth factor, and t_D is the *doubling-time*. Each locality can have its own $\{N_o,\, t_D,\, t = 0\}$ values, and K_D and t_D should be nearly constant during this initial period of CoVID-19 spread.

Standard epidemiology identifies the number of people N_G a known infected individual had recent contact with. Contacts of that N_G group are tracked next, followed by additional tracking stages. This process sets the K_D value.

Society-wide *Mitigation Measures* such as: (a) *Social Distancing*, (b) wearing face masks in public, (c) prohibiting large gatherings, (d) implementing large-scale population testing, (e) disinfecting high-touch surfaces in public areas, (f) enhanced cleaning of items brought into homes, and (g) minimizing human contact with likely virus-containing materials and matter; all can help reduce $N(t)$ growth. These *Mitigation Measures* can modify the Eq. [1.1a] epidemiology model by causing the local t_D values to lengthen.

In order to model these *Mitigation Measures*, t_D and K_D become explicit functions of time, $t_D(t)$ and $K_D(t)$. Using a linear function for $t_D(t)$ lengthening is one of the simplest time-varying extensions. A linear function of time also corresponds to the first term of a *Taylors' Series* expansion of some more general $t_D(t)$ analytic function, giving this epidemiology extension:

$$t_D(t) \equiv t_{dbl}\,(1 + \alpha_S\, t)\,, \qquad\qquad\qquad [1.3a]$$
$$K_D(t) \equiv \tfrac{(\ln 2)}{t_D(t)} = (\ln 2)\,/\,[t_{dbl}\,(1 + \alpha_S\, t)] \equiv K_o\,/\,(1 + \alpha_S\, t)\,. \qquad [1.3b]$$

The $t = 0$ initial values for $t_D(t)$ and $K_D(t)$ become the new constants t_{dbl} and K_o, which characterize the initial exponential growth phase. The α_S coefficient in Eq. [1.3a] is a new *Social Mitigation Parameter [SMP]* that helps quantify the effectiveness of the society-wide *Mitigation Measures* as a whole.

The α_S value expresses how well non-infected people manage to avoid the virus contagion. As a lumped parameter, it likely reflects an average value over many processes, known and unknown, which comprise mitigation, to supplement the contact-to-contact tracking that initially sets t_{dbl} or K_o.

Substituting Eq. [1.3b] into Eq. [1.1a] gives:

$$N(t) = N_o \exp[+K_o\, t\,/\,(1 + \alpha_S\, t)]\,, \qquad\qquad [1.4]$$

as one of the simplest models for CoVID-19 spread. A pure exponential growth (or decay) has no *memory*, while Eq. [1.4], for $\alpha_S > 0$, has a *memory*. The $t = 0$ start time of first mitigation changes the future history. To include $t < 0$ requires replacing Eq. [1.3a] by $t_D(t) \equiv t_{dbl}\,(1 + \max[0,\, \alpha_S\, t\,])$, which has a corner at $t = 0$ that preserves the *memory* of when mitigation first started.

2   Model Features

The Eq. [1.1a] exponential growth pandemic model implicitly assumes a large uninfected population allows the disease to easily spread ("*dilute pandemic*"). When almost everybody is infected ("*saturated pandemic*"), exponential growth shuts off, and Eq. [1.1a] no longer applies.

On 3/10/2020, *German Chancellor Angela Merkel*[1] noted that she "*estimates that 60% to 70% of the German population will contract the coronavirus*", indicating that *saturated pandemic* models are being considered as a worst-case.

Even that worst-case condition assumes: (i) recovered coronavirus patients are no longer infectious, and (ii) surviving a CoVID-19 infection confers absolute immunity to re-infection. Recently, South Korea[2] found 91 cases of clinically recovered patients later testing as CoVID-19 positive. They may also shed viable coronaviruses in their phlegm and fecal matter[3], furthering disease spread. Although these effects are not modeled here, those additional transmission modes could turn a 60%-70% hope into a 99+% consequence.

These factors show why CoVID-19 modeling beyond Eq. [1.1] is needed, especially to see if society-wide *Mitigation Measures* can naturally halt disease spread, without necessitating a *saturated pandemic* condition. We show next that Eq. [1.4] allows for this pandemic shut-off, even in the *dilute pandemic* case. Since both K_o and α_S in Eq. [1.4] have the same units, their ratio is dimensionless. The long-term limit of Eq. [1.4] gives:

$$\underset{t \to +\infty}{Lim}\left[N(t)\,/\,N_o\right] = \exp[+K_o\,/\,\alpha_S\,], \qquad\qquad [2.1]$$

setting an average value for the total number of all follow-on infections arising from a single individual. Since it depends only on the ratio of the original pandemic growth factor K_o to the α_S *SMP*, this model shows the impact of accounting for a broader environment beyond individual contact tracking.

The early spread of CoVID-19 cases outside of China[4], and the early USA CoVID-19 data[5] both had nearly exponential rises, as shown in **Figure 1**. A purely exponential rise gives a straight line on a *log-plot* {*log(# of cases) vs linear time*}. The initial *doubling-time* for the USA was $t_{dbl} \approx 2.02\ days$, giving $K_o \approx 0.343\,/\,day$ using Eqs. [1.1a]-[1.1b]. This initial CoVID-19 data was prior to any significant *Mitigation Measures* being implemented.

On March 19, 2020, *Governor Gavin Newsom* of California ordered a CoVID-19 "stay-at-home" lockdown of virtually all of California's ~40 million residents. Similar statewide CoVID-19 lockdowns were ordered by the Governors of Illinois, New York, Indiana, Michigan, Ohio, Washington, West Virginia and Wisconsin.

The slowing of CoVID-19 spread by implementing large-scale societal *Mitigation Measures* can be fairly rapid, as illustrated by the USA CoVID-19 data of **Figure 2**, which covers March 2020.

The impact of these multi-state *Mitigation Measures* is evident in **Figure 2** as a sudden transition on the *log-plot* from a straight-line to having downward curvature, which the α_S *Social Mitigation Parameter (SMP)* aims to quantify. The local slope in **Figure 2** also decreases right after the onset of *Mitigation Measures*, indicating further slowing of CoVID-19 spread.

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A well-documented South Korean coronavirus cluster can also be used to help estimate the expected size of the α_S *SMP*. That CoVID-19 cluster determined that a single infected person at the *Shinjeongji Church* caused infection of about $4,482$ people[4] within the 47-day time interval between January 20, 2020 and March 8, 2020. Those data provide this α_S estimate:

$$\ln(4,482) \approx (0.343\,/\,day)\,(47\,days)\,/\,[\,1 + \alpha_S\,(47\,days)\,]\,, \qquad [2.2a]$$
$$\alpha_S = 0.01952\,/\,day \approx 0.02\,/\,day, \qquad [2.2b]$$

as indicative of minimal mitigation. If additional deliberate mitigation measures doubled α_S to $\alpha_S \approx (0.04\,/\,day)$, Eq. [2.1] would give:

$$\underset{t \to +\infty}{Lim}\,[N(t)\,/\,N_o] = \exp[\,+\tfrac{(0.343/day)\,(47\,days)}{1+(.04/day)\,(47\,days)}] \approx 270\,, \qquad [2.3]$$

for the number of infections per person, a $16.6\,X$ reduction from $4,482$.

Since $N(t)$ in Eq. [1.4] represents a total number of cases, it is similar to a *cumulative distribution function (cdf)*, which is used in reliability and also has time as its fundamental variable. The derivative of Eq. [1.4], $dN(t)\,/\,dt$, is analogous to an unnormalized *probability density function (pdf)*, which can be used to predict a *pandemic peak*:

$$\{pdf\} \equiv \tfrac{d}{dt}[\,\tfrac{N(t)}{N_o}\,] = \tfrac{d}{dt}\{\exp[\,+K_o\,t\,/\,(1+\alpha_S\,t)\,]\} =$$
$$+K_o(\tfrac{1}{1+\alpha_S\,t})^2\,\exp[\,+K_o\,t\,/\,(1+\alpha_S\,t)\,]\,, \qquad [2.4a]$$
$$\{pdf\} = +K_o\,[\,\tfrac{N(t)}{N_o}\,]\,/\,(1+\alpha_S\,t)^2, \qquad [2.4b]$$

The time t_P of the *pandemic peak* is set by:

$$\tfrac{d}{dt}\{pdf\} \equiv 0\,, \qquad [2.5a]$$
$$\alpha_S\,t_P = \tfrac{1}{2}[K_o/\alpha_S] - 1\,, \qquad [2.5b]$$

where Eq. [2.5b] simplification arises from the Eq. [2.5a] constraint. Substituting $K_o = 0.343\,/\,day$ and $\alpha_S = 0.02\,/\,day$ from Eq. [2.2b] into Eq. [2.5b] gives $t_P \approx 379\,days$. Increasing mitigation to $\alpha_S = 0.04\,/\,day$, keeping the same $K_o = 0.343\,/\,day$, now gives $t_P \approx 82\,days$, which is a ˜$4.6\,X$ reduction in the *pandemic peak* timing for doubling $(2X)$ the mitigation effect from its original baseline value. These examples highlight the tremendous impact that even a small amount of enhanced social mitigation can have.

While the $\alpha_S = 0$ limit of Eq. [2.4b] recovers the Eq. [1.1a] standard exponential growth, both the $\{pdf\}$ and $[N(t)/N_o]$ growth are then unbounded. However, even a small $\alpha_S > 0$ value in Eq. [2.4b] will have an enormous impact on the predicted long-time behavior. Since Eq. [2.1] showed that $[N(t)/N_o]$ now approaches a finite value for all $\alpha_S > 0$, this new $\{pdf\}$ asymptotic limit:

$$\underset{t \to +\infty,\ \alpha_S > 0}{Lim}[\{pdf\}] \sim \tfrac{\{\text{Constant}\}}{(\alpha_S\,t)^2}\,, \qquad [2.6]$$

also results. The $\{pdf\}$ that arises from this model all have an initial exponential rise, coupled with the Eq. [2.6] "*long tail*" at large times, which means that new CoVID-19 cases may arise for a long time, even if significant *Mitigation Measures* are in place.

The Eq. [2.6] $\{pdf\}$ prediction also differs substantially from the widely-used *University of Washington IHME* (*Institute for Health Metrics and Evaluation*) projections, which use symmetric Gaussians for both the $\{pdf\}$ rise and fall[6]. Thus, these methods provide an alternative risk-bound for evaluating potential CoVID-19 worst-case scenarios.

3   Determining $\{t_{dbl},\ \alpha_S\}$ from CoVID-19 Data

Explicit numerical values for $\{t_{dbl},\ \alpha_S\}$ parameters were determined from the CoVID-19 data as follows. Rewriting Eq. [1.4] as:

$$\ln[N(t)\,/\,N_o] = [+K_o\,t\,/\,(1+\alpha_S\,t\,)\,] \equiv (\ln 2)\,t\,/\,[t_{dbl}(1+\alpha_S\,t\,)\,]. \qquad [3.1]$$

allowed data fitting to be done on a $Y - vs - X$ *log-plot*, using $Y = \ln[N(t)\,/\,N_o]$ as the ordinate and $X \leftrightarrow t$ as the abscissa, to calculate and minimize the *root-mean-square (rms) error*.

The $t = 0$ point in Eq. [3.1] sets N_o. To best model *Mitigation Measures*, this point was usually chosen at the start of a downward curvature on a *log-plot*, so that $N(t = 0) \equiv N_I$, where the N_I is now the first data point in the analysis. The prior $t < 0$ regime can often have a nearly pure exponential growth, as in **Figure 2**, and those regions should not be part of *rms-error* minimization for evaluating *Mitigation Measures*.

The N_F final data point, measured at the most recent $t = t_F$ time:

$$N(t = t_F) \equiv N_F\,. \qquad [3.2]$$

was also fixed for each dataset, so that only $\{t_{dbl},\ \alpha_S\}$ value pairs that meet both $N(t = 0) \equiv N_I$ and Eq. [3.2] were used.

In practice, an α_S was chosen first. The $Excel^{TM}\_Tools\_Goal\text{-}Seek$ function was used to adjust t_{dbl} to obey Eq. [3.2], setting the *rms-error* between the dataset and Eq. [3.1], with the final $\{t_{dbl},\ \alpha_S\}$ having the least *rms-error*.

In the following figures, all CoVID-19 raw data came from the publicly available Microsoft^{TM} "*COVID-19 Tracker*" site[7]. When no updates were available, that site repeated the prior day data, whereas we used the geometric mean of the day-prior and day-after data for interpolation.

4   Effects of Varying the Initial Zero-Time Point

Starting with:

$$N(t) = \mathbf{1}\,\exp[+K_o\,t\,/\,(1+\alpha_S\,t\,)\,+\ln(N_o)]\,, \qquad [4.1a]$$

$$\underset{t\to+\infty}{Lim}\,[N(t)\,] = \mathbf{1}\,\exp[+(K_o\,/\,\alpha_S)+\ln(N_o)\,]\,, \qquad [4.1b]$$

$$t_{dbl}^o \equiv (\ln 2)\,/\,K_o\,, \qquad [4.1c]$$

using a shifted time-scale normalization point is examined next:

$$N_S(t') = \mathbf{1}\,\exp[+K_o\,(t'+t_A)\,/\,[1+\alpha_S\,(t'+t_A)]+\ln(N_o)]\,, \qquad [4.2]$$

This $N_S(t')$ function should closely match Eq. [4.1a], with a shifted time axis: $t = t'+t_A$, but the best fit parameter numerical values change. Since:

$$[1+\alpha_S\,(t'+t_A)] = [1+\alpha_S\,t_A]\,[1+\tfrac{\alpha_S\,t'}{(1+\alpha_S\,t_A)}]\,, \qquad [4.3a]$$

$$\tfrac{K_o\,(t'+t_A)}{[1+\alpha_S\,(t'+t_A)]} = [1+\tfrac{t_A}{t'}]\,\tfrac{(K_o\,t')}{[1+\alpha_S\,t_A]}\,/\,[1+\tfrac{\alpha_S\,t'}{(1+\alpha_S\,t_A)}]\,, \qquad [4.3b]$$

then defining:

$$\alpha_A \equiv \alpha_S/(1+\alpha_S\,t_A)\,, \qquad [4.4a]$$

$$K_A \equiv K_o/(1+\alpha_S\,t_A)\,, \qquad [4.4b]$$

$$t_{dbl}^A \equiv (\ln 2)\,/\,K_A = (1+\alpha_S\,t_A)\,t_{dbl}^o = (1+\alpha_S\,t_A)\,(\ln 2)\,/\,K_o\,, \qquad [4.4c]$$

it gives:

$$\tfrac{K_o\,(t'+t_A)}{[1+\alpha_S\,(t'+t_A)]} = [1+\tfrac{t_A}{t'}]\,K_A\,t'\,/\,[1+\alpha_A t']$$

$$= K_A\, t' \,/\, [1 + \alpha_A t'] + K_A\, t_A \,/\, [1 + \alpha_A t'] \,. \qquad [4.5]$$

These equations highlight the net effect of time-shifting. For $t_A > 0$, when t' begins after *Mitigation Measures* have started, the shifted-time-axis results in a larger calculated *doubling-time* and a smaller *SMP* α_A-value. For $t_A < 0$, when t' may include *Mitigation Measures* already in place at the Eqs. [4.1a]-[4.1c] $t = 0$ point, this shifted-time-axis results in a smaller calculated *doubling-time* and a larger *SMP* α_A-value.

Finally, for small t', where $\alpha_A t' < 1$, using Eqs. [4.1b] and [4.5] gives:

$$\mathbf{1}\,\exp[\tfrac{+K_o\, t}{(1+\alpha_S\, t)} + \ln(N_o)] \approx \mathbf{1}\,\exp[\tfrac{+K_A\, t'}{(1+\alpha_A\, t')} + \ln(N_A)]\,, \qquad [4.6a]$$

$$\ln(N_A) = \ln(N_o) + K_A\, t_A\,, \qquad [4.6b]$$

which shows that the $t' = 0$ new initial state should have an N_A starting value obeying $N_A > N_o$ for $t_A > 0$, and $N_A < N_o$ for $t_A < 0$. However, whether $\{N_o,\, K_o,\, \alpha_S\}$, or an alternative $\{N_A,\, K_A,\, \alpha_A\}$, are used to parameterize a given data set, the net overall function fit and predictions, as a function of calendar date, should remain fairly self-consistent, even when some ambiguity exists as to when *Mitigation Measures* first were noticeably effective.

5 USA and Selected States Model Results

The model predictions for CoVID-19 spread in the USA is shown in **Figure 3**. This analysis only included data after mid-March 2020, when several State Governors first instituted mandatory *Mitigation Measures*. Results give an *SMP* estimate of $\alpha_S \approx 0.5945/day$, a USA initial *doubling-time* of $t_{dbl}^{initial} \approx 2.1758\ days$, which lengthens to $t_{dbl}^{\{at\ Peak\}} \approx 5.83\ days$ at the projected *pandemic peak* of ˜4/19/2020. The predicted total number of CoVID-19 cases is ˜$5,464,000$, giving a projected ˜1.67% final infection rate, if the present level of *Mitigation Measures* or their equivalent, are continued.

These predictions assume no "*second wave*" of infection or re-infection. They also do not include the effect of additional *Mitigation Measures*, which could further increase the $\{t_{dbl},\ \alpha_S\}$ values, and significantly reduce the projected final number of CoVID-19 cases.

Figure 4 shows model predictions for CoVID-19 evolution in California. Only data after 3/21/2020 was included in the analysis, after California Governor Gavin Newsom instituted mandatory *Mitigation Measures*. It gives an *SMP* estimate of $\alpha_S \approx 0.03546/day$, with an initial *doubling-time* of $t_{dbl}^{initial} \approx 2.5017\ days$, which lengthens to $t_{dbl}^{\{at\ Peak\}} \approx 9.774\ days$ at the projected *pandemic peak* of ˜6/07/2020. The predicted total number of CoVID-19 cases is ˜$1,123,700$, giving a projected ˜2.813% final infection rate, at the present level of *Mitigation Measures*.

Figure 5 shows model predictions for CoVID-19 evolution in New York. A relatively high *SMP* estimate of $\alpha_S \approx 0.1031/day$ was found, coupled with a relatively short initial *doubling-time* of $t_{dbl}^{initial} \approx 0.9395\ days$, which creates a high narrow spike in daily new cases. The present model projects a New York *pandemic peak* around 4/10/2020, with an estimated at-peak *doubling-time* of

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$t_{dbl}^{\{at\ Peak\}} \approx 3.36\ days$. The predicted total number of cases at ˜1,218,000, giving a projected ˜6.072% final infection rate.

Figure 6 shows model predictions for CoVID-19 evolution in Washington State. An initial *doubling-time* of $t_{dbl}^{initial} \approx 2.189\ days$ and an *SMP* value of $\alpha_S \approx 0.0687/day$ were found, with a projected *pandemic peak* around 6/04/2020. The relatively low number of cases at the *Mitigation Measures* start helps to give a predicted total number of cases of ˜557,600, corresponding to a ˜7.15% final infection rate.

Figure 7 shows model predictions for CoVID-19 evolution in Illinois. An initial *doubling-time* of $t_{dbl}^{initial} \approx 2.457\ days$ and moderate *SMP* value of $\alpha_S \approx 0.0373/day$ combine to give a projected *pandemic peak* around 6/04/2020, similar to Washington State, but having a higher predicted total number of cases at ˜1,277,000, and a projected ˜11.47% final infection rate.

Figure 8 shows model predictions for CoVID-19 evolution in Florida. Many Florida counties instituted their own Mitigation Measures prior to a state-wide lockdown, slowing CoVID-19 growth. A somewhat high *SMP* value of $\alpha_S \approx 0.0526/day$, and an initial *doubling-time* of $t_{dbl}^{initial} \approx 1.494\ days$ results. A *pandemic peak* is estimated at around 5/20/2020, with a predicted total number of cases at ˜1,090,000, and a projected ˜4.96% final infection rate.

6 World and Selected Countries Model Results

Figure 9 shows model predictions of CoVID-19 evolution for the whole World. The present-day *doubling-time* value of $t_{dbl}^{initial} \approx 5.761\ days$ likely represents a combination of small urban, large urban, and rural area results. However, the calculated low *SMP* estimate of $\alpha_S \approx 0.01712/day$ shows that nearly ˜4.43% of the World's population could be at risk for eventual CoVID-19 infection. At these present levels, the projected *pandemic peak* is around 8/15/2020, with potentially hundreds of millions of people being infected.

Figure 10 shows model predictions for CoVID-19 evolution in China, covering their "*first wave*" of early exposure and early mitigation. Data were included that was prior to a "*New Reporting Method*" being used, which started off with one sudden data jump, and nearly level CoVID-19 follow-on results. The present model predicts what number of cases could have resulted, had the reporting method not changed. Draconian *Mitigation Measures* helped to contain the pandemic to *Hubei Province* and *Wuhan*. These projections show that those *Mitigation Measures* have impressively contained CoVID-19 spread.

Figure 11 shows model predictions for CoVID-19 evolution in South Korea, covering the period of their country's early exposure and initial mitigation methods. Pre-pandemic *Mitigation Measures*, including extensive contact-tracing and large-scale CoVID-19 testing, were implemented. These projections show that those *Mitigation Measures*, as an alternative to China's methods, also have impressively contained CoVID-19 spread.

Figure 12 shows model predictions for CoVID-19 evolution in Italy. An initial *doubling-time* of $t_{dbl}^{initial} \approx 1.4648\ days$ and *SMP* estimate of $\alpha_S \approx$

$0.05282/day$ give a *pandemic peak* around $4/29/2020$, with a predicted number of total cases at $\tilde{}1,764,000$, and a projected $\tilde{}2.92\%$ final infection rate.

Figure 13 shows model predictions for CoVID-19 evolution in Germany. The relatively high *SMP* estimate of $\alpha_S \approx 0.07614/day$ with an initial *doubling-time* of $t_{dbl}^{initial} \approx 1.4177\ days$ combine to give a projected *pandemic peak* at around $4/08/2020$, with a predicted total number of cases of $\tilde{}700,100$, and a projected $\tilde{}0.84\%$ final infection rate. These values would make Germany one of the less impacted countries in Europe. They represent predicted final CoVID-19 infection rates that are significantly lower than the original $60\% - 70\%$ early worst-case estimates highlighted by *German Chancellor Angela Merkel*.

Figure 14 shows model predictions for CoVID-19 evolution in Spain. An *SMP* estimate of $\alpha_S \approx 0.07058/day$, which is comparable to Germany, and a smaller initial *doubling-time* of $t_{dbl}^{initial} \approx 1.1778\ days$ combine to give more predicted CoVID-19 cases than Germany. The estimated *pandemic peak* is around $4/21/2020$, with a predicted number of total cases at $\tilde{}1,526,000$, and a projected $\tilde{}3.26\%$ final infection rate.

Figure 15 shows model predictions for CoVID-19 evolution in Ecuador. Reports of chaos in Ecuador have been alarming. Yet the present data show a significant and somewhat unexpected leveling off in the number of reported CoVID-19 cases. This result could mean that some as yet unknown *Mitigation Measures* may be operating. Alternatively, the data could mean that there is a dire CoVID-19 testing and reporting shortfall operating amidst the chaos.

Figure 16 shows model predictions for CoVID-19 evolution in India. These initial data show virtually no mitigation at present, having one of the lowest calculated *SMP* estimates of $\alpha_S \approx 0.0148/day$, with an initial *doubling-time* of $t_{dbl}^{initial} \approx 3.135\ days$. At this rate, nearly 17.38% of the population of India could eventually become infected. The estimated *pandemic peak* is around $5/30/2021$, which would be 441 days after the first CoVID-19 fatality was reported, on $3/14/2020$. Additional *Mitigation Measures*, further increasing the $\{t_{dbl},\ \alpha_S\}$ values, as well as adding in additional modeling parameters may significantly reduce these projected number of CoVID-19 cases.

7 Augmented Peak Shape Modeling

Using a new *Social Mitigation Parameter [SMP]* α_S, as in Eq. [1.3a], successfully models pandemic shut-off, even in the *dilute pandemic* limit. However, as the **Figures 3-16** insets show, many of the data-vs-model comparisons have the data trending above the model near the final $t = t_F$ data point.

Since Eq. [1.3a] for $t_D(t)$ is linear, using an *Additional Modeling Parameter [AMP]* β_S in a higher order polynomial, may fit the $\{pdf\}$ shape better. A quadratic function for t_D:

$$t_D = t_{dbl}\left(1 - \beta_S\, Z + \alpha_S\, Z^2\right), \qquad\qquad [5.1]$$

where $N(t)/N_o$ still approaches a constant at long times, as in Eq. [2.1a], then sets $Z^2 \equiv t$, giving this extension of Eq. [1.3a]:

$$t_D(t) = t_{dbl}\left(1 - \beta_S\,\sqrt{t} + \alpha_S\, t\right). \qquad\qquad [5.2]$$

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Values of $\beta_S > 0$ in Eq. [5.2] allow the predicted $[N(t)/N_o]$ values to rise above the $\beta_S = 0$ model predictions, and to have a smaller *doubling-time*, for the same $\{t_{dbl}, \alpha_S\}$. However, the best fit $\{t_{dbl}, \alpha_S\}$ values will also differ between the $\beta_S > 0$ and $\beta \equiv 0$ cases, so these changes are relative.

The new $\{pdf\}$ function for Eq. [5.2] is:

$$\{pdf\} \equiv \frac{d}{dt}[\frac{N(t)}{N_o}] = \frac{d}{dt}\{\exp[+K_o\,t/(1 - \beta_S\,\sqrt{t} + \alpha_S\,t)]\} =$$

$$+K_o\,[\frac{N(t)}{N_o}]\,(1 - \tfrac{1}{4}\beta_S^2\,t)\,/\,[(1 + \alpha_S\,t)^2\,(1 + \tfrac{1}{2}\beta_S\,\sqrt{t})]\,. \qquad [5.3]$$

When $\{pdf\} = 0$ in Eq. [5.3], it estimates an end-point for the pandemic at:

$$t_{END} = (4/\beta_S^2)\,, \qquad [5.4]$$

while predicting this maximum number of pandemic cases at t_{END}:

$$N(t) = N_o\,\exp[K_o/(\alpha_S - \tfrac{1}{4}\beta_S^2)]\,. \qquad [5.5]$$

An an example, this augmented model was applied to CoVID-19 evolution in Italy. As shown in **Figure 17**, this Eq. [5.3] $\{pdf\}$ function gives a better fit to the observed number of daily new CoVID-19 cases.

The initial *doubling-time* of $t_{dbl}^{initial} \approx 2.5566\,days$, along with estimates for the *Mitigation Measure* parameters of $\alpha_S \approx 0.04583/day$ and $\beta_S \approx -0.1725$, in this augmented model, combine to significantly reduce the projected maximum number of CoVID-19 cases down to ˜264, 820, which is an ˜7X less compared to using α_S alone, as in **Figure 12**. This augmented model sets an estimated *pandemic peak* at 3/29/2020, with a projected pandemic end-point around 7/7/2020, which is also significantly more optimistic.

The true CoVID-19 pandemic progress is likely to be in between **Figure 12** as a worst-case, and **Figure 17** as a best-case projection. The geometric mean of the **Figure 12** and **Figure 17** results set an average of ˜683, 500 cases for Italy at the CoVID-19 pandemic end. These bounds also highlight the amount of uncertainty that is intrinsic to these empirically based methods.

8   Summary and Conclusions

The standard exponential for modeling pandemics starts with an N_o known number of initial cases at some reference time $t = 0$. Epidemiologists work to determine a pandemic growth factor K_D, which sets the *doubling-time* t_D for the number of pandemic cases.

The CoVID-19 disease, caused by the SARS-CoV-2 coronavirus pathogen, initially showed both regional and global exponential growth. It resulted in a *doubling-time* of $t_D \approx 2.02\,days$ for the US, as highlighted in **Figure 1**.

An exponential growth normally only halts when it runs out of materials. In epidemiology that point often occurs when there are virtually no more uninfected people left, which we call a *saturated pandemic*. The exponential growth function is only applicable when infection rates are much lower than saturation, which we call a *dilute pandemic*.

A modification to exponential growth is developed here, which allows ratio of the number of pandemic cases, $N(t)$, compared to its N_o initial value at $t = 0$:

$$\underset{t \to +\infty}{Lim}\,[N(t)/N_o] = \mathbf{M}\,, \qquad [6.1]$$

to approach a final constant, denoted **M**, while still being in a *dilute pandemic* condition. This result is attributed to the inclusion of society-wide *Mitigation Measures* to stop pandemic growth, before the value of **M** reaches the whole population value.

Society-wide *Mitigation Measures* aim to progressively lengthen the t_D *doubling-time*, essentially making $t_D(t)$. Most analyses presented here used a linear function of time as a simplest non-constant model for $t_D(t)$:

$$t_D(t) \equiv t_{dbl}\,(1 + \alpha_S\,t)\,, \qquad\qquad\qquad [6.2a]$$

$$K_D(t) \equiv \tfrac{(\ln 2)}{t_D(t)} = (\ln 2)\,/\,[t_{dbl}\,(1 + \alpha_S\,t)] \equiv K_o\,/\,(1 + \alpha_S\,t)\,, \qquad [6.2b]$$

where $t_D(t = 0) \equiv t_{dbl}$, and $K_D(t = 0) \equiv K_o$. Here, α_S is a new *Social Mitigation Parameter (SMP)*, to quantify societal *Mitigation Measures*. This Eq. [6.2a] extension of a pure exponential growth gives:

$$N(t) = N_o\,\exp[+K_o\,t\,/\,(1 + \alpha_S\,t)]\,, \qquad\qquad [6.3]$$

as an empirical equation for modeling CoVID-19 spread. Since both K_o and α_S in Eq. [1.4] have the same units, their ratio is a dimensionless number. The long-term limit of Eq. [6.3] gives:

$$\underset{t \to +\infty}{Lim}\,[N(t)\,/\,N_o] = \exp[+K_o\,/\,\alpha_S\,]\,, \qquad\qquad [6.4]$$

setting a final value for the Eq. [6.1] constant **M**, allowing these predictions to be applicable to the *dilute pandemic* limit. The CoVID-19 number of estimated cases per day is given by:

$$\{pdf\} = \tfrac{d}{dt}[N(t)/N_o]\,, \qquad\qquad\qquad [6.5a]$$

$$\underset{t \to +\infty,\,\alpha_S > 0}{Lim}[\{pdf\}] \sim \tfrac{\{\text{Constant}\}}{(\alpha_S\,t)^2}\,, \qquad\qquad [6.5b]$$

which combines an initial exponential rise with "*long tail*" at large times. In this model, new CoVID-19 cases can continue to arise for a long time, even with significant *Mitigation Measures* in place.

Analysis of available CoVID-19 data using this model shows that it can match observed data fairly well, both from various US states [**Figures 3-8**], as well as for different global countries [**Figures 9-17**]. However, using a single parameter to encompass all societal *Mitigation Measures* often gives a slightly larger slope on a *log-plot*, compared to the latest measured data values, which makes this model a likely worst-case estimate.

A second data-fitting parameter β_S was also used in an augmented model, to better fit the $\{pdf\}$ data:

$$N(t) = N_o\,\exp[+K_o\,t\,/(1 - \beta_S\,\sqrt{t} + \alpha_S\,t)\,]\,, \qquad [6.6a]$$

$$\max\,[N(t)\,/\,N_o] = \exp[+K_o\,/\,(\alpha_S - \tfrac{1}{4}\beta_S^2)\,]\,, \qquad [6.6b]$$

$$t_{END} = (4\,/\,\beta_S^2)\,, \qquad\qquad\qquad [6.6c]$$

where t_{END} becomes an estimated pandemic end-point, where zero new CoVID-19 cases per day could occur.

As a representative example, this augmented model was applied to the CoVID-19 data from Italy [**Figure 17**]. Those results show that this augmented model allows a better fit to the observed number of new daily CoVID-19 cases, but the absence of a CoVID-19 tail in its $\{pdf\}$ function makes this $\{K_o,\,\alpha_S,\,\beta_S\}$ augmented model a likely best-case result, with the original $\{K_o,\,\alpha_S\}$ model being a likely worst-case estimate.

This class of CoVID-19 pandemic models all enable pandemic shut-off even in the *dilute pandemic* limit, with only a small fraction of the total population being infected. These models also provide estimates for: (a) the maximum number of cases near pandemic shutoff, (b) the size and shape of the *pandemic peak* $[dN(t)\,/\,dt]$, and (c) pandemic peak timing $[t_P]$. These models and analyses may help enhance planning and preparation to maximize resource use, potentially increasing individual and collective CoVID-19 pandemic survival rates.

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more gradual function for the Daily New CoVID-19 cases, making these predictions a likely worst-case.

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Figure 15: Predicted ECUADOR CoVID-19 results. Reports of chaos in Ecuador have been alarming. Poor CoVID-19 tracking and low testing may have skewed these results.

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Figure 17: Predicted ITALY CoVID-19 results, using an augmented 2-parameter $\{\alpha_S, \beta_S\}$ *Social Mitigation* model. Total number of CoVID-19 cases is much less than **Figure 12**, but the model post-peak drop is much steeper, making this a likely best-case result.

10   References

1.   https://www.nytimes.com/2020/03/11
          /world/europe/coronavirus-merkel-germany.html
2.   https://www.yahoo.com/news/recovered-coronavirus-patients-
          test-positive-161747102.html
3.   https://www.scmp.com/news/hong-kong/health-environment
          /article/3075609/coronavirus-can-be-transmitted-through-faeces
          {South China Morning Post/Hong Kong/Health and Environment}
4.   https://medium.com/@tomaspueyo
          /coronavirus-act-today-or-people-will-die-f4d3d9cd99ca; "*Chart 2*".
5.   https://en.wikipedia.org/wiki
          /2020_coronavirus_pandemic_in_South_Korea; "*Timeline - March*".
6.   https://www.MedRxiv.org/ ID=MEDRXIV/2020/043752v1, 03.25.2020,
          "*Forecasting COVID-19 impact on hospital bed-days, ICU-days,
          ventilator-days and deaths by US state in the next 4 months*",
          IHME COVID-19 Health Service Utilization Forecasting Team.
7.   https://www.bing.com/covid?form=CPVD07.

Early Stages of CoVID-19 Growth Outside of China
Initial USA Data shows Doubling Time of ~2.02 Days

(a) Total Cases of Coronavirus Outside of China

Source: Tomas Pueyo, based on worldometers data

(b) Initial CoVID-19 Cases in the US
Doubling Time: t/dbl\ ~ 2.02 days

◆ USA-Total   ── Pure-Exponential
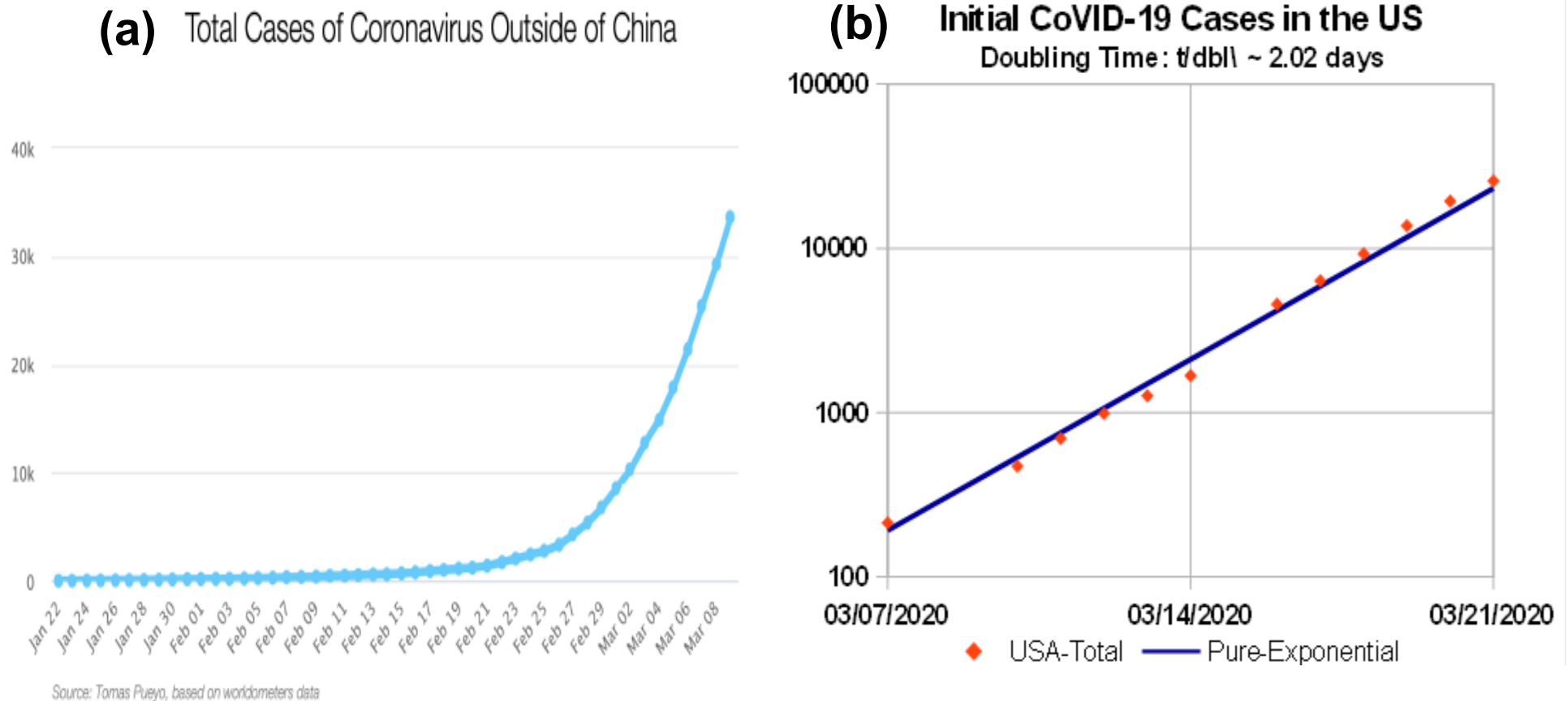
Left Side: Chart 2 from 3/11/2020 article *"Coronavirus: Why You Must Act Now"* by Thomas Pueyo
https://medium.com/@ThomasPueyo/coronavirus-act-today-or-people-will-die-f4d3d9cd99ca
Right Side: Initial Number of USA CoVID-19 Cases, Bing.com Coronavirus Daily Compilations

**Figure 1: Early CoVID-19 Cases: (a) Outside of China, and
(b) Just for the US. Both graphs show nearly exponential growth.**

Societal *Mitigation Measures* Slows Down CoVID-19 Growth
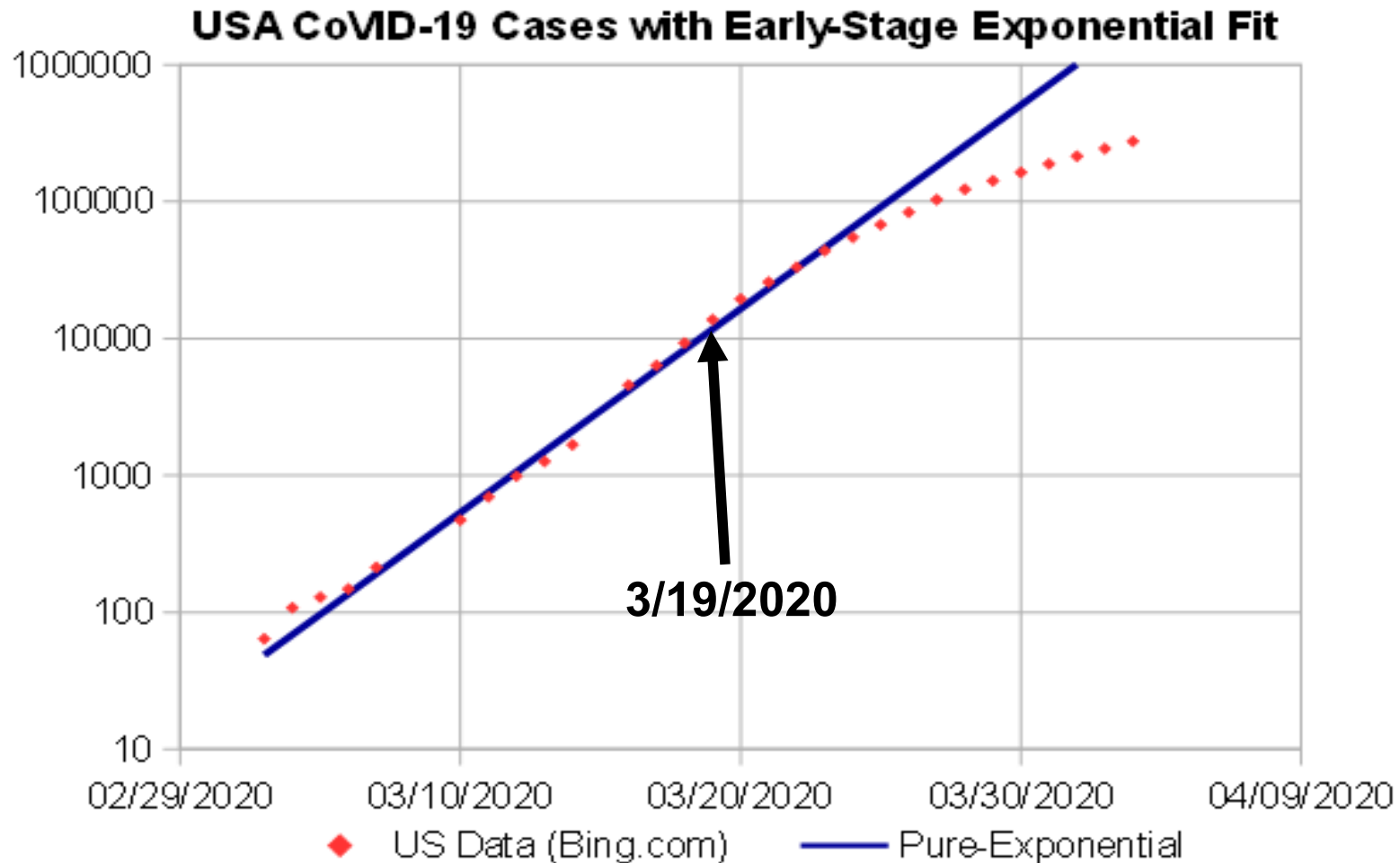
- 3/16-19/2020: CA Gov. Newsom Orders Closure of Schools followed by full lockdown of all Non-Essential Business; other states soon follow.

- CoVID-19 growth slows post-3/19/2020 Closures

Figure 2: USA CoVID-19 data, pre-vs-post mid-March 2020. Multi-State *Mitigation Measures* slowed growth, transitioning from straight-line to downward curvature.
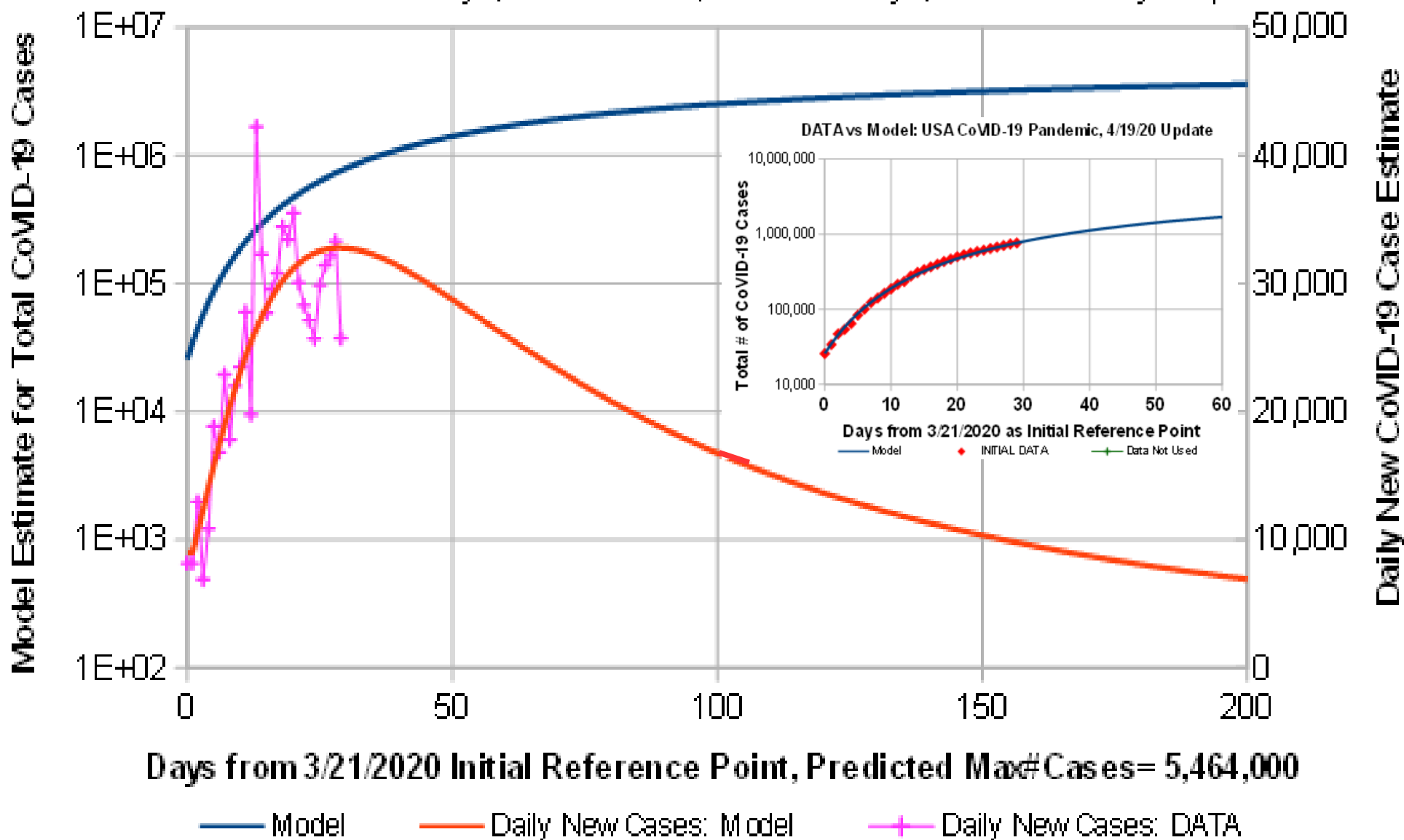
Figure 3: USA Model Predictions. To allow better mitigation predictions, only data after mid-March 2020 was included, when several Governors instituted mandatory lockdowns.

Figure 4: Predicted California CoVID-19 results. After Gov. Gavin Newsom instituted widespread *Mitigation Measures*, projections showed significant improvement.
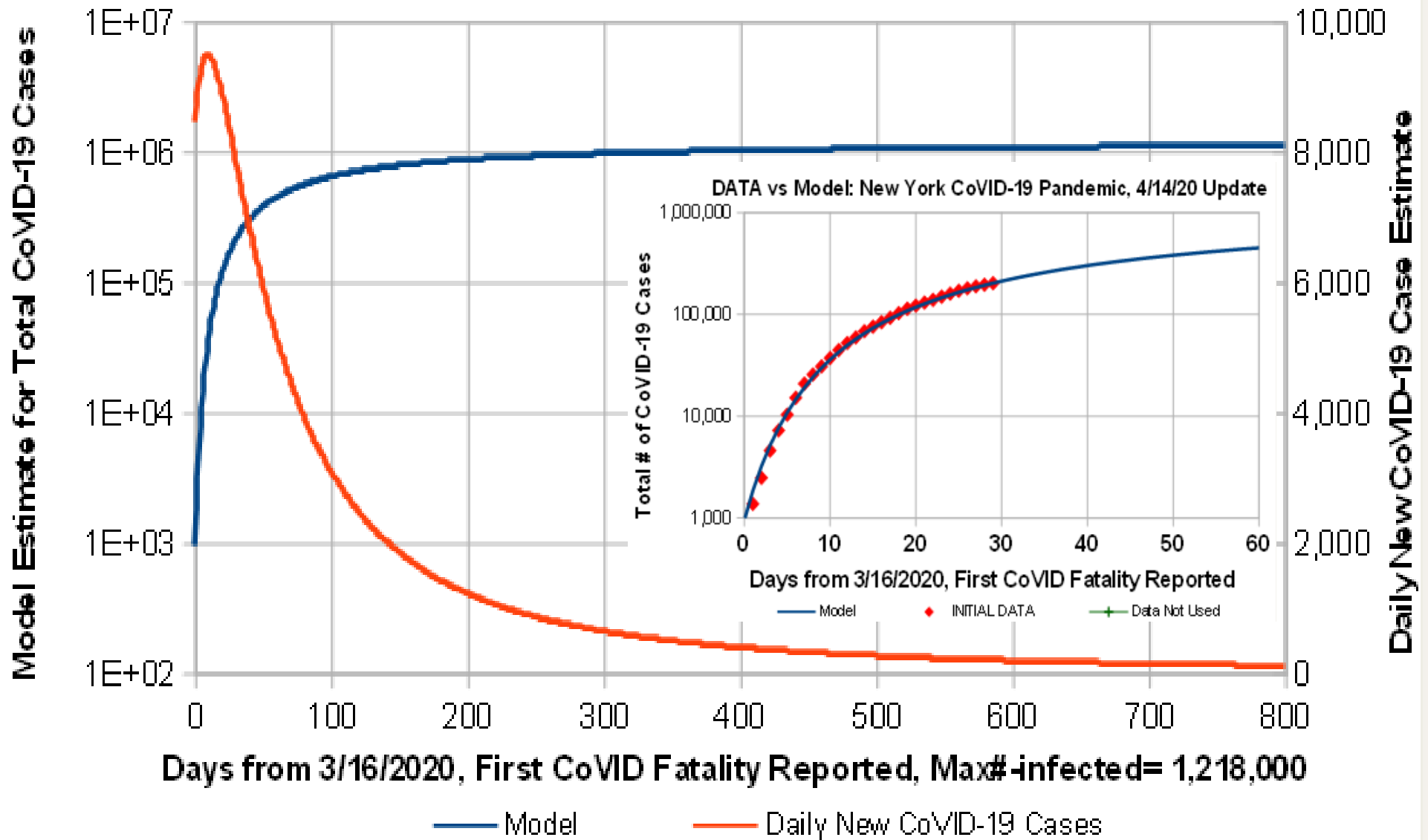
Figure 5: Predicted New York CoVID-19 results. A relatively high *Mitigation Measure* level and a short intrinsic doubling time creates a narrow spike in daily new cases.

Figure 6: Predicted Washington State CoVID-19 results. The relatively low number of cases at _Mitigation Measure_ start helps to give a relatively low final number of cases.
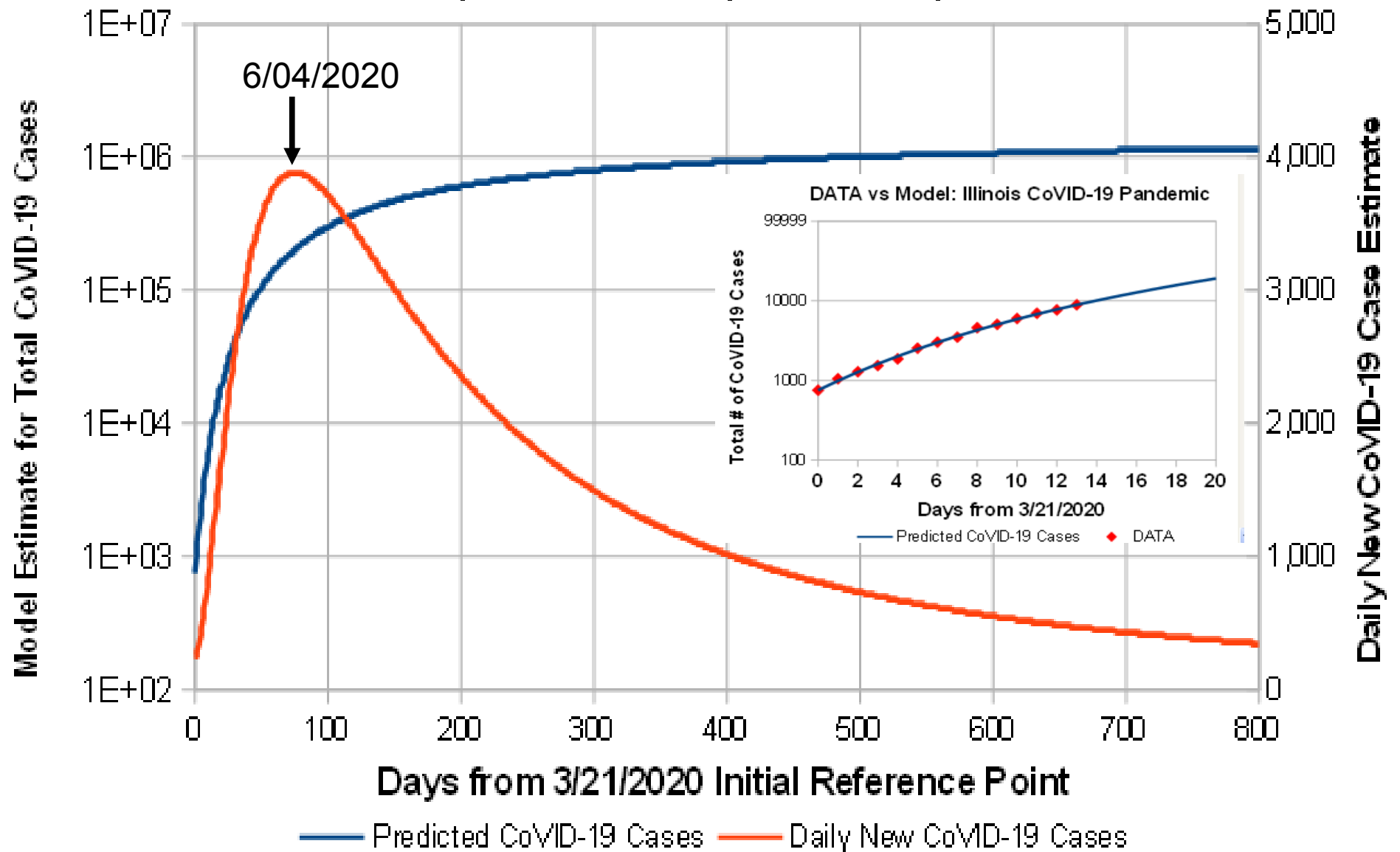
Figure 7: Predicted Illinois CoVID-19 results. The slow doubling time and moderate amount of *Mitigation Measures* gives a slow increase to the predicted CoVID-19 peak.
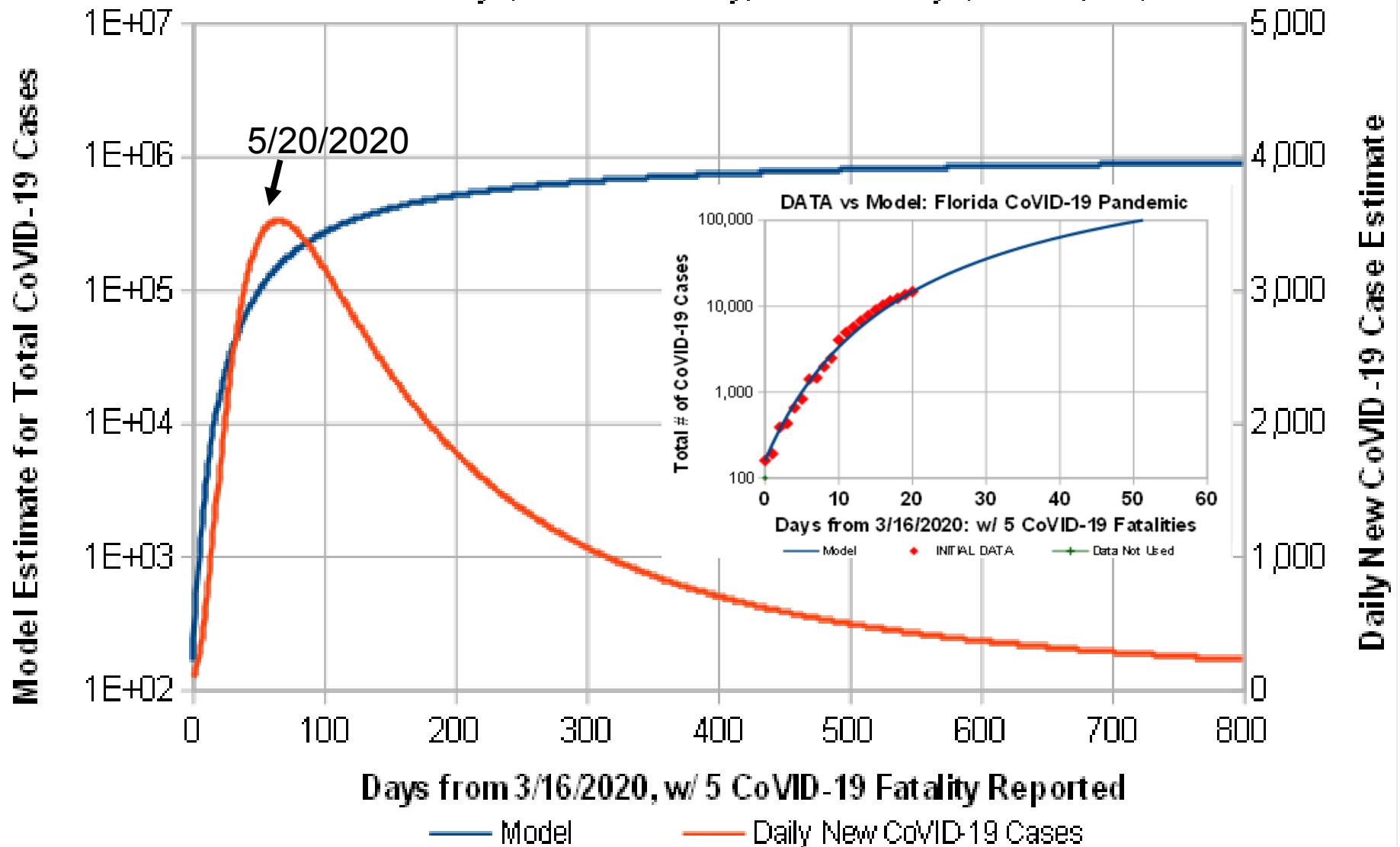
Figure 8: Predicted Florida CoVID-19 results. Many Florida counties instituted their own *Mitigation Measures* prior to a state-wide lockdown, substantially slowing CoVID-19 growth.
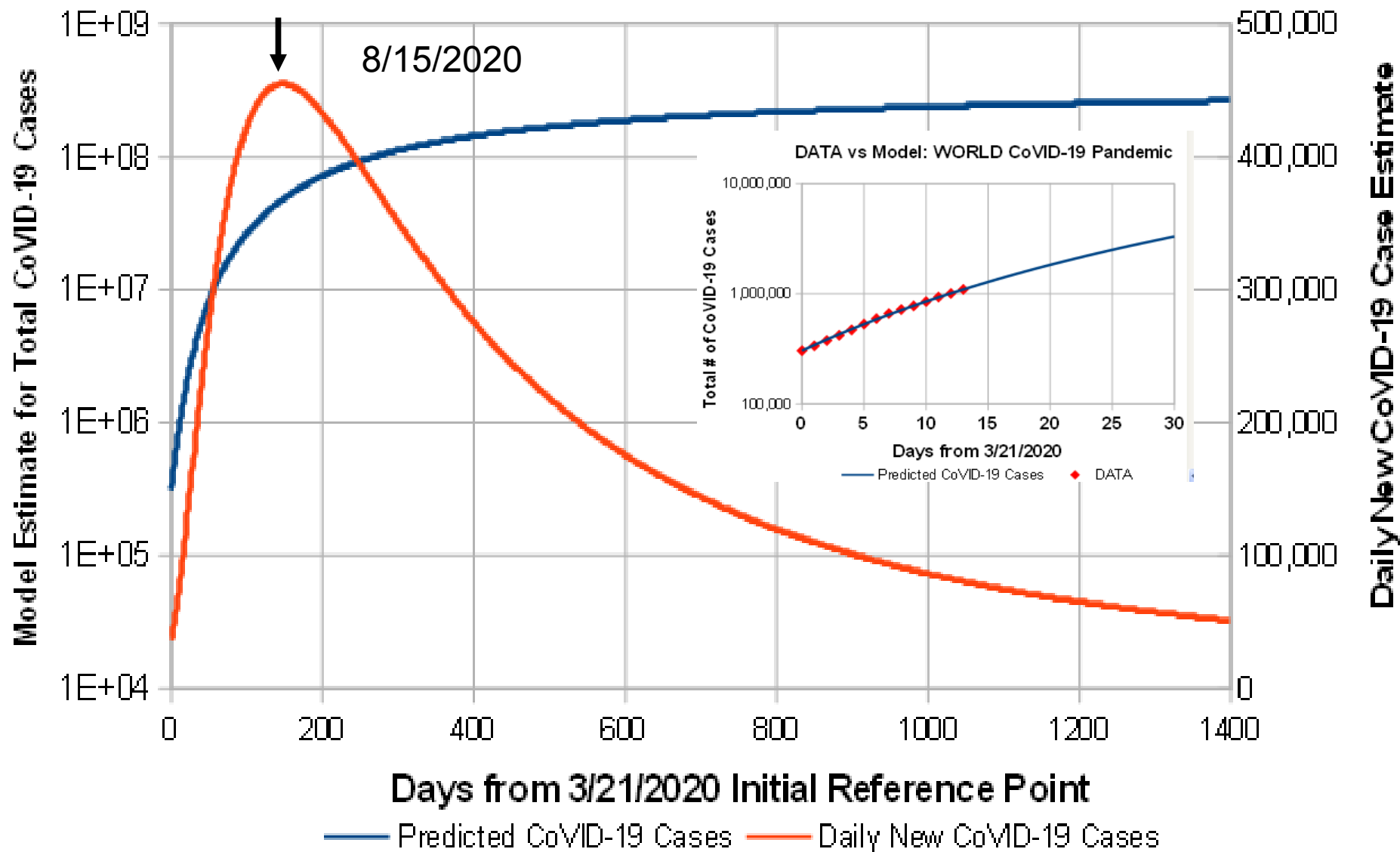
Figure 9: Model predictions for the WORLD, showing present-day low level of mitigation.
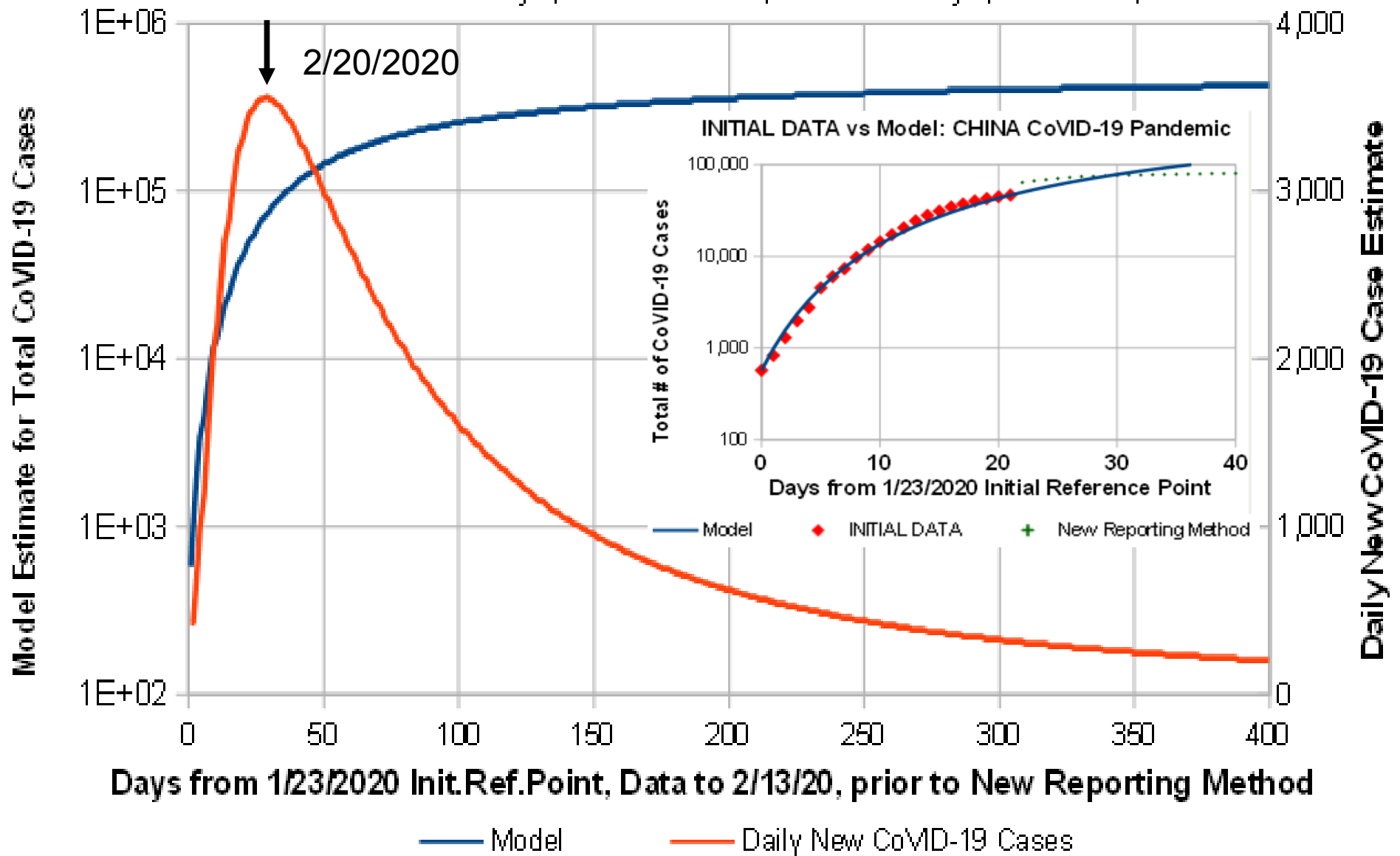
Figure 10: Predicted CHINA CoVID-19 results, using pre-"*New Reporting Method*" data. Draconian *Mitigation Measures* helped to contain pandemic to *Hubei Province* and *Wuhan*.
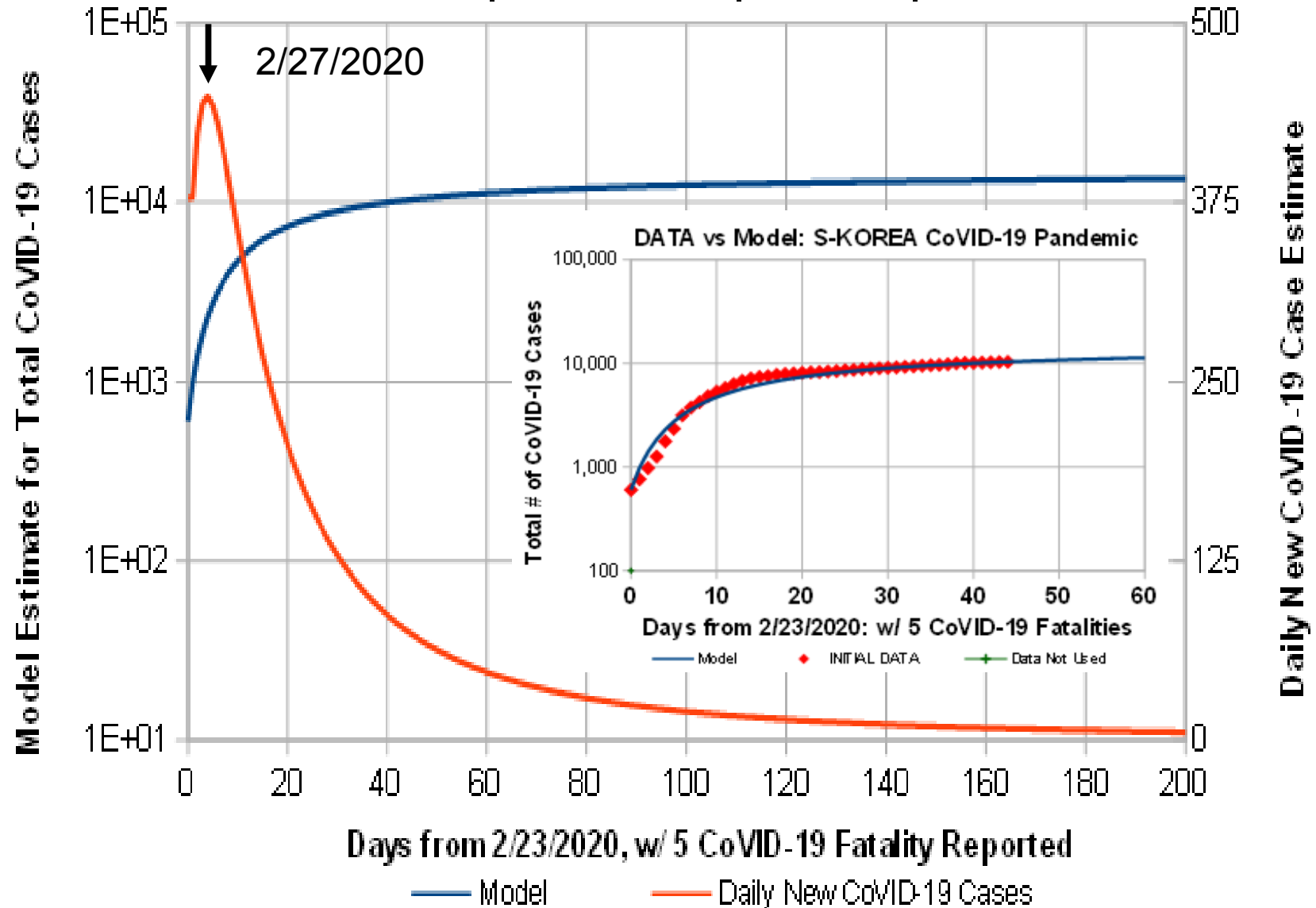
Figure 11: Predicted SOUTH KOREA CoVID-19 results. Pre-pandemic contact-tracing and large-scale CoVID-19 testing as *Mitigation Measures* have contained the pandemic.
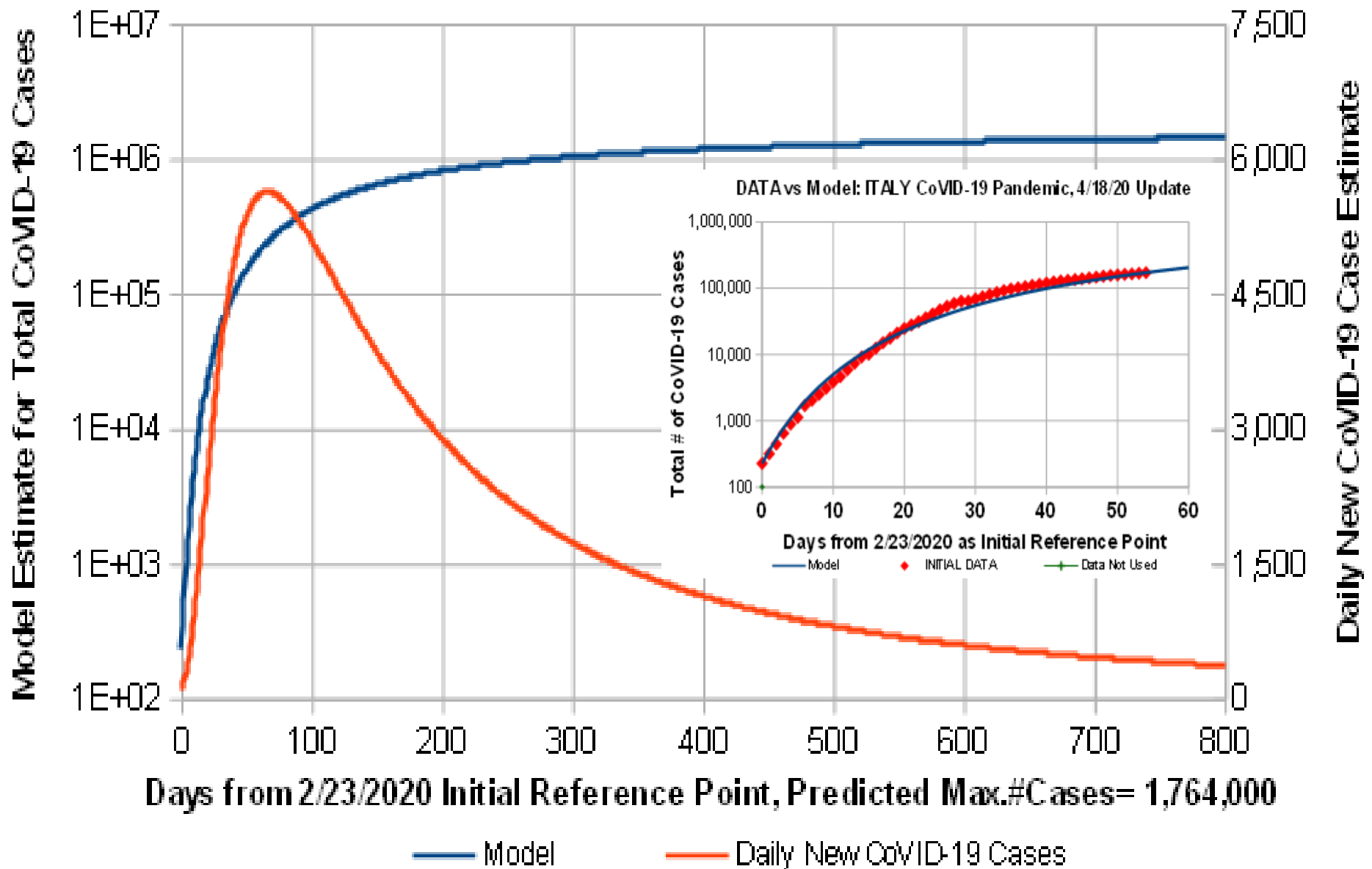
Figure 12: Predicted ITALY CoVID-19 results. Additional curvature in the actual CoVID-19 data vs Model makes these predictions a likely worst-case.
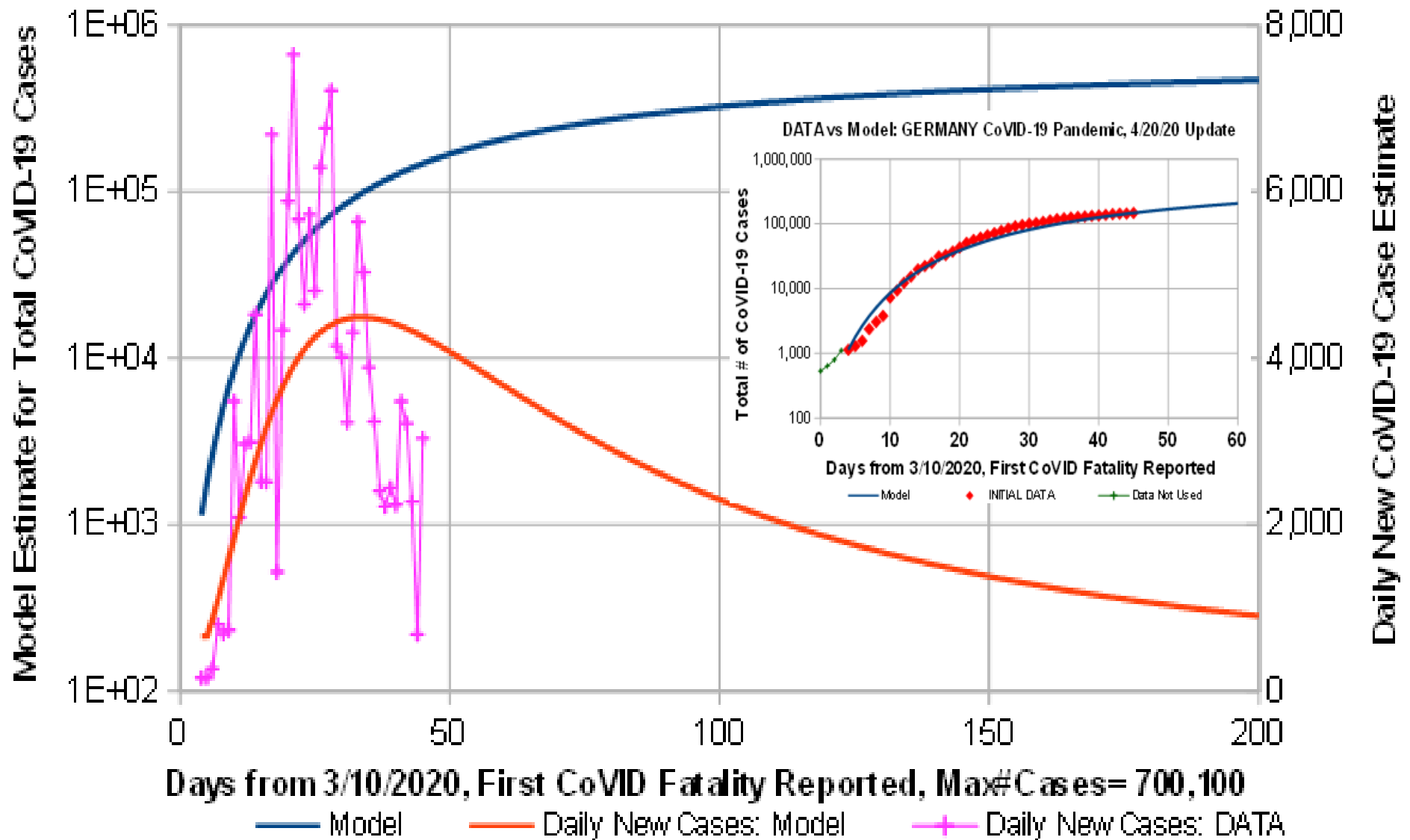
Figure 13: Predicted GERMANY CoVID-19 results. This model gives a more gradual function for the Daily New CoVID-19 cases, making these predictions a likely worst-case.

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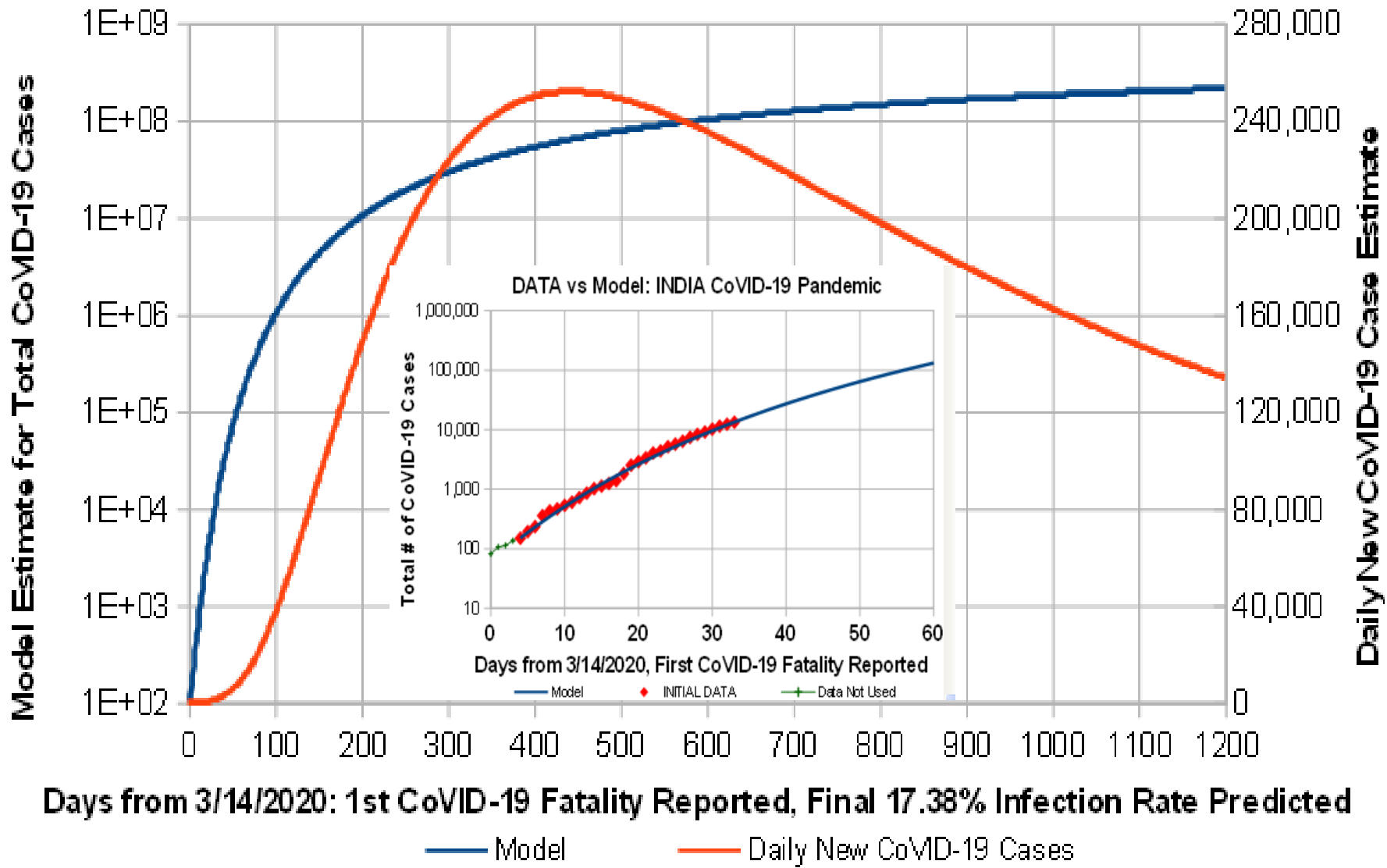
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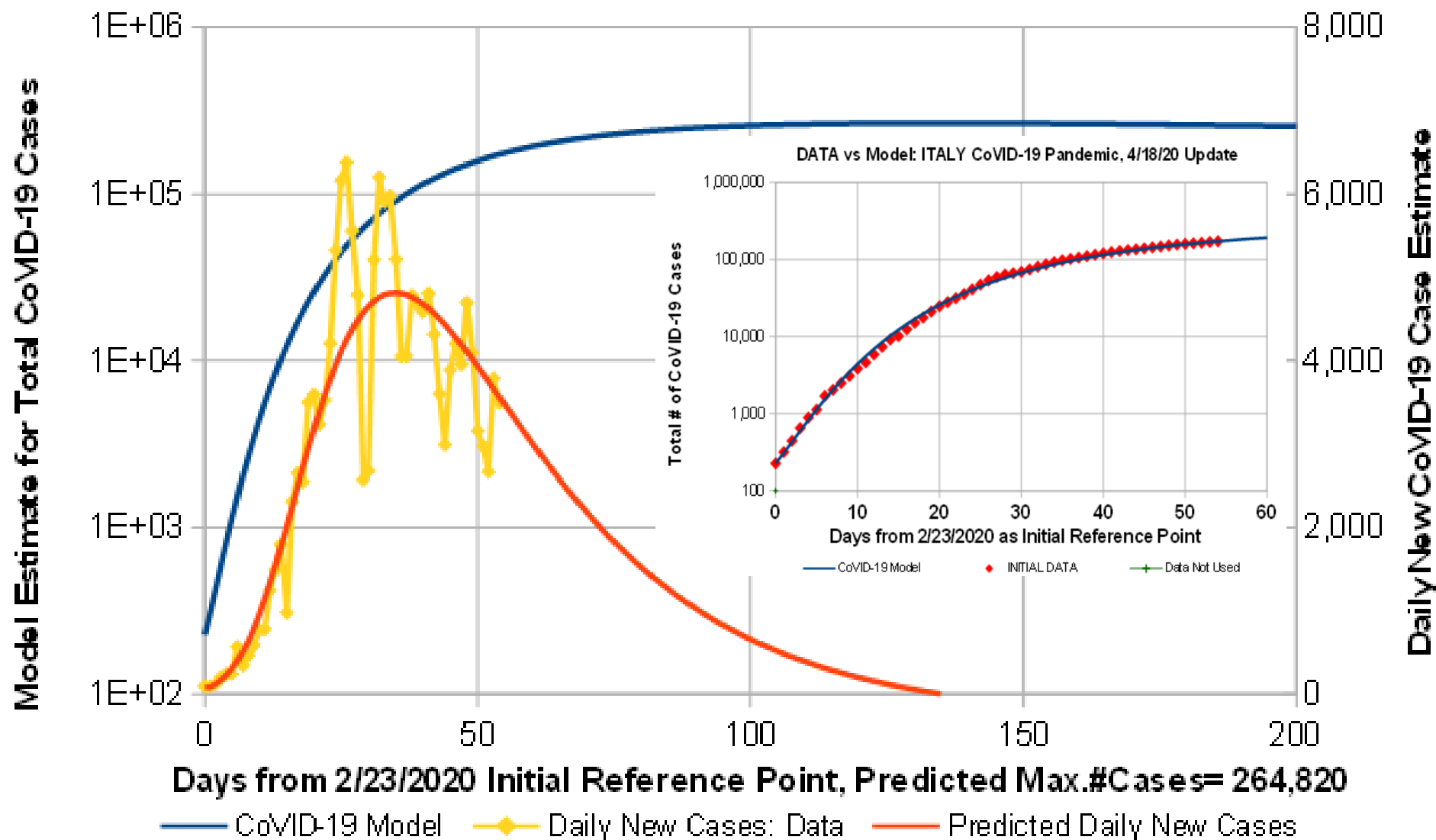
Figure 17: Predicted ITALY CoVID-19 results, using an augmented 2-parameter {α/s\, β/s\} *Social Mitigation* **model. Total number of CoVID-19 cases is significantly less than Fig. 12, but the model post-peak drop is much steeper, making this a likely best-case result.**