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How effective is vaccination protection?

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Summary

This short note provides a simple static model to assess the effect of vaccination against hospitalization.

1 Introduction.

The number of vaccinees admitted to hospitals can give us a false picture of the effectiveness of vaccination. Indeed, the proportion of hospitalized vaccinees is often expected to equal the proportion of vaccinees in the population. Two limiting cases always hold: if no one is vaccinated, 100% of hospital admissions are unvaccinated; if everyone is vaccinated, 100% of hospital admissions are vaccinated. In general, however, estimating the proportion of hospitalized vaccinees is not so straightforward.

In the following, a very simple static model will be given that can be used to assess the effectiveness of vaccination against hospitalization, i.e., a more severe course of the disease. As a measure of effectiveness, we choose the ratios between hospitalized unvaccinated to vaccinated in the population.

2 Model.

Let P be the population size, $x \in [0,1]$ the proportion of vaccinated people, and $1-x$ the proportion of unvaccinated. Let H be the number of hospitalized, $y \in [0,1]$ the proportion of vaccinated hospitalized, and $1-y$ the proportion of hospitalized unvaccinated. We assume that the number of vaccinated hospitalized is proportional to the number of vaccinated people. Then we can write

$$yH = a \cdot xP, \quad (1)$$

$$(1-y) \cdot H = b \cdot (1-x) \cdot P, \quad (2)$$

where a and b are the fractions of hospitalized vaccinated and unvaccinated in the population, respectively. The ratio between (2) and (1) is

$$c \equiv \frac{b}{a} = \frac{x(1-y)}{y(1-x)}. \quad (3)$$

We take this ratio as a measure of vaccination effectiveness. Vaccination will be effective if $c > 1$, i.e., $b > a$, the chance of being hospitalized if unvaccinated is higher than if vaccinated.

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To determine a and b, we add (1) and (2), and introduce the fraction α, i.e., the fraction of people been hospitalized,

$$\alpha \equiv \frac{H}{P} = a + b. \tag{4}$$

Taking into account (3), we get

$$a = \frac{\alpha}{1+c}, \tag{5}$$

$$b = \frac{\alpha c}{1+c}. \tag{6}$$

If we know c and assume it is a constant, then the proportion of hospitalized vaccinated equals

$$y = \frac{x}{c + (1-c)\cdot x}. \tag{7}$$

An example of a graph of this function is shown in Figure 1. The proportion of hospitalized vaccinated and unvaccinated will be the same when the proportion of vaccinated is $y = 1/2$. Now (7) yields

$$x_* = \frac{c}{1+c} \tag{8}$$

We note that the assumption on constant c is artificial and cannot generally hold. In fact, the vaccination effectiveness c is not constant, as can be seen from the graph in Figure 4, which shows the dependence of c on the proportion of fully vaccinated persons in the population for the course of the Covid-19 epidemic in Slovenia.

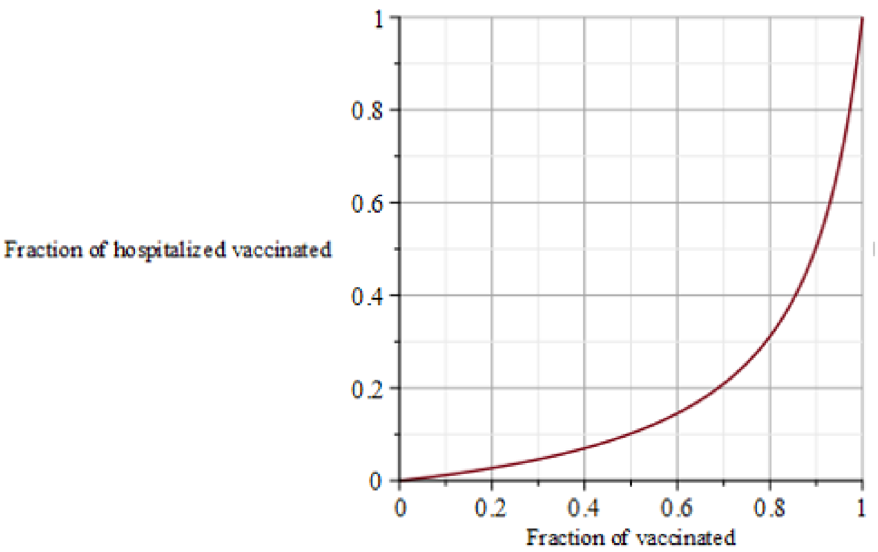

Figure 1: Graph of the proportion of hospital admissions according to the formula (8) for the case when $c = 0.9$

3 Example

As a practical example, let us look at the calculation of vaccination effectiveness for the course of the Covid 19 epidemic in Slovenia in 2021. The data used are published on the Tracker website [2]

Graphs 2 and 3 show the cumulative proportion of vaccinated adults in the population and among hospitalized patients.

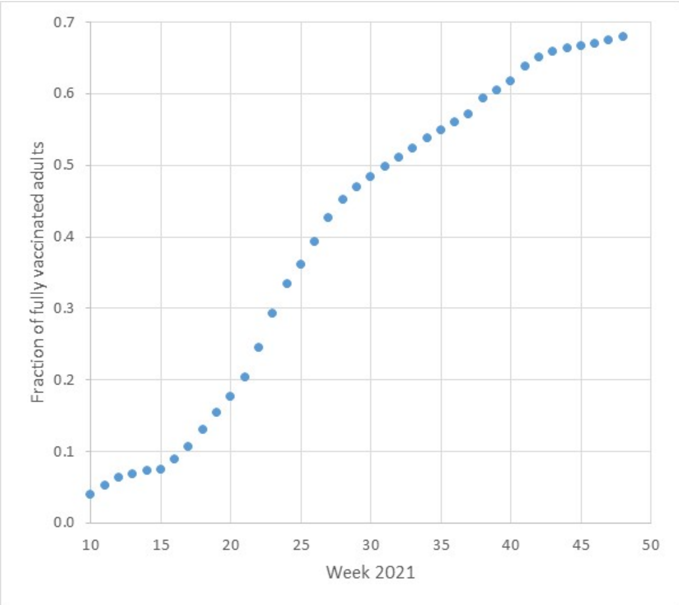

Figure 2: Proportion of fully vaccinated adults during the Covid-19 epidemic in Slovenia

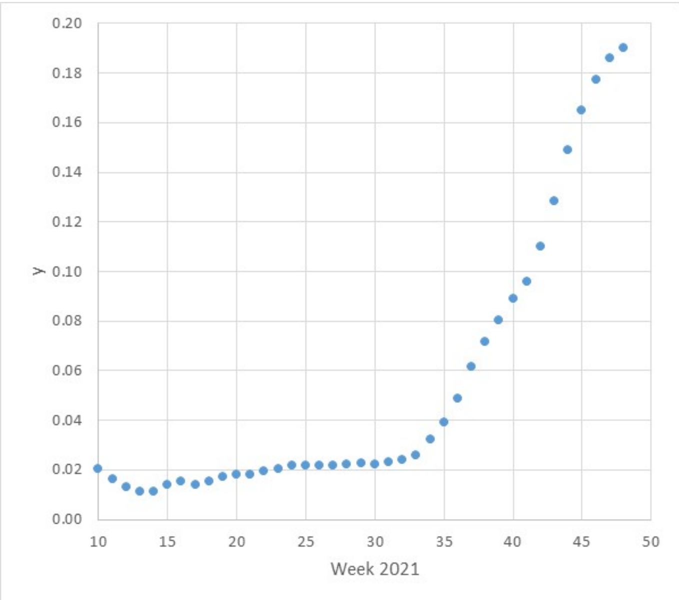

Figure 3: Proportion of hospital admissions with complete vaccination during the Covid-19 epidemic in Slovenia.

[2] https://covid-19.sledilnik.org/en/stats

Currently (Dec 2021), 68% of the adult population in Slovenia is vaccinated and the proportion of hospitalized vaccinated is currently 19%. Thus, for $x = 0.68$ and $y = 0.19$ we obtain, using (3),

$$c = \frac{0.68 \times 0.81}{0.19 \times 0.32} \approx 9.03 \tag{9}$$

For the given data, this means that so far, for everyone vaccinated hospitalized person, about 9 unvaccinated people have been hospitalized.

If the above ratio were to be maintained, we would end up with a ratio of 1:1 in hospitals when the number of vaccinated

$$x_* = \frac{9.03}{1 + 9.03} \approx 0.90 \tag{10}$$

i.e., about 90%.

However, as already mentioned, vaccination effectiveness is not constant, as shown by the graph in Figure 4, which shows vaccination effectiveness against the proportion of fully vaccinated adults. In terms of time, the effectiveness of protection was highest somewhere around week 30, i.e., the beginning of August, when the proportion of vaccinated adults was about 50%, and the hospitals had only a few patients. With the new epidemic wave, the effectiveness started to drop, but in the last few weeks, it has stabilized at somewhere around 9.

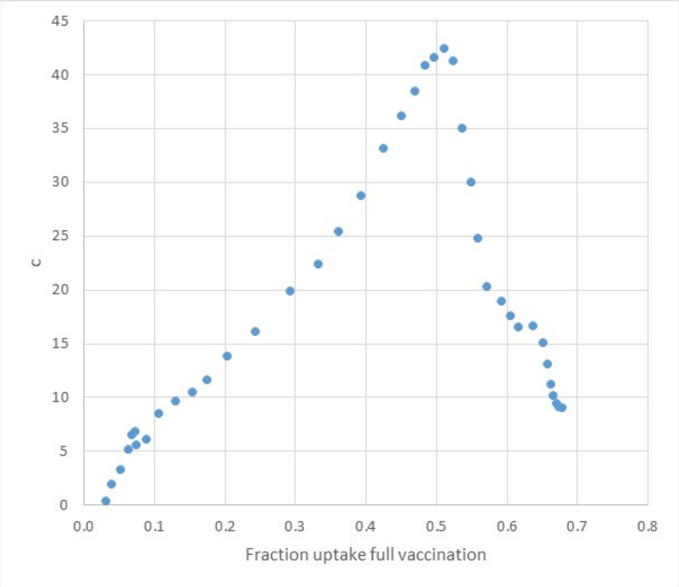

Figure 4: Calculated vaccination effectiveness c during the Covid-19 epidemic in Slovenia from March 2021 (week 9) to December 2021 (week 48).

On Graph in Figure 5, the fractions of nonvaccinated and vaccinated population is shown, where for calculation we use Eq. (5) and (6). On the graph, one can easily observe that the population ratio reaches 9:1.

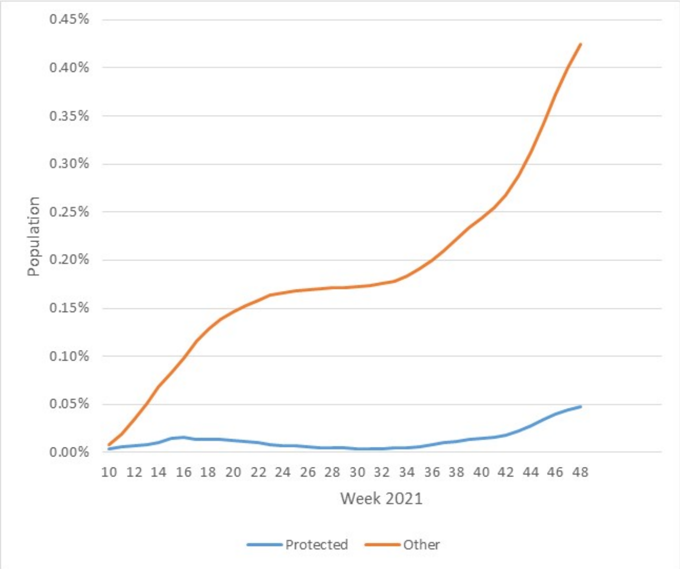

Figure 5. Fraction of fully vaccinated adults and others for Covid 19 epidemic in Slovenia.

4 Conclusion

This short paper provides a simple static model that can be used to assess the effectiveness of vaccination protection, i.e., the ratio between hospitalized vaccinated and unvaccinated in population. The example shown is for hospitals, but it can easily be applied to the case of confirmed cases or victims of an epidemic.