Initial stage of the COVID-19 infection process in human population

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The simplest approximation for the first stages of the infection spread is considered. The specific feature of the COVID-19 characterized by its long latent period is taken into account. Exponential increase of numbers of infected people is determined by the half period of the maximal latent time for the COVID-19. The averaging over latent period leads to additional increase of the infected numbers.

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I. INTRODUCTION

The new virus named COVID-19 appeared first in China in December 2019 and to this moment (17 March 2020, 11:29 GMT) led to 186665 confirmed cases of disease and 7467 deaths in the World. The percent of the deathes is unusually high, especially in the group of old people, in comparison with the other flu infection associated with other viruses (see the website https://www.worldometers.info/coronavirus/coronavirus-cases/). 11 March 2020 this disease is recognized by the World Health Organization (WHO) as a pandemic. We consider the simplest model of the COVID-19 spread which takes into account a long latent period of this disease.

There are various scenario and models for the description of the epidemic situation. The most models are concentrated on the spontaneous development of the infection spread to describe all stages of the process. An example is, e.g., the susceptible-infected-susceptible (SIS) - one of the basic themes in the mathematical epidemiology [1,2]. These models describe the balance between the susceptible and infected individuals in population under the various conditions of infection transfer [3]. Kinetic approach can be applied on the basis of works [4], [5]. However, the situation with the COVID-19 has the specific features which should be taken into account.

There are essential efforts to confront the epidemic, to localize its spread. In general we can characterize its development as a non-spontaneous epidemic process. It is especially important also to slow down the propagation velocity to enable medicine to provide treatment for patients. Under these conditions the investigation of the initial stage of infection is needed. The long latent period of the COVID-19 is the feature which is considered in this paper.

II. THE INFECTION DEVELOPMENT

If the number of dangerous (at distance less $s \simeq 2$ meters) contacts per day equals N_c for one infected person and k is the coefficient of infection transfer (the ratio of the number of people accepting virus to the number of dangerous contacts N_c) we find that the average number of infected persons in population after l_0 days equals the half of the maximal latent time $2l_0$ days for the COVID-19.

$$N_i(N_c; k; l_0) = (N_c \cdot k + 1)(N_c \cdot k)^{l_0-1}, \tag{1}$$

where l_0 is the average number of days between obtaining the virus and appearance of the illness after which persons have to be isolated.

As an example, let us consider $N_c = 50$, $k = 0.1$ and $l_0 = 7$ (based on the half of the maximal latent period equals of 14 days). As easily seen, in this example $N_i(50; 0.1; 7) = 93750$. If $N_c = 25$, $k = 0,1$ and $l_0 = 7$ we find $N_i(25; 0.1; 7) = 854$. The value of k parameter is unknown. It can be roughly estimated but practically cannot be changed. The average value $l = l_0$ is well known, but of course, there is some distribution on l, which is not included in the simplest model under consideration. In Fig.1 the dependence of the infected cases
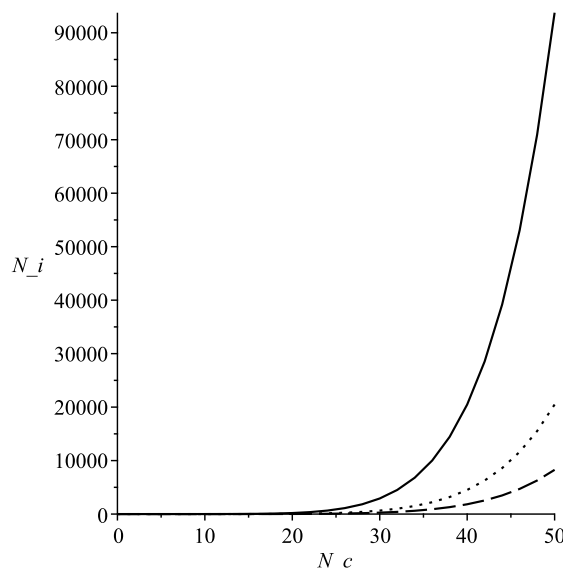
Figure 1: Infected cases N_i versus the dangerous contacts N_c for the different coefficient of infection transition k. The solid line corresponds to $k = 0.1$, the dot line to the value $k = 0.08$ and the dashed line $k = 0.07$ after $l_0 = 7$ equals the half period of the maximal latent time.

N_i on the number of dangerous contacts N_c per one infected person is shown in the typical interval $50 \geq N_c \geq 0$. The various curves correspond to the coefficient of infection transfer (or probability of infection) equals $k = 0.1, 0.08, 0.07$.

To generalize the above consideration for the case of distribution on l, which always exists, we use the Gauss normalized distribution around l_0 (the restriction for the maximal latent period is removed due to exponential decrease of the distribution)

$$\sqrt{\frac{4D}{\pi}} \frac{\exp[-D(l - l_0)^2]}{1 + \Phi(\sqrt{D}\, l_0)} \tag{2}$$

where Φ is the error function $\Phi(x) = [2 \int_0^x dx' \exp(-x'^2)]/\sqrt{\pi} = -\Phi(-x)$. To find approximately the result for the average on l value of infected people $\langle N_i(N_c; k; l)\rangle_l$ we can transfer to the continuous description on l and calculate the integral ($a \equiv N_c \cdot k$)

$$\langle N_i(N_c; k; l)\rangle_l = \left(1 + \frac{1}{N_c \cdot k}\right) \int dl W_G(l) a^l = \frac{\sqrt{4D}}{\sqrt{\pi}[1 + \Phi(\sqrt{D}\, l_0)]} \int_0^\infty dl a^l \exp[-D(l - l_0)^2] \tag{3}$$

Using the stationary-phase method we arrive at the approximate result for the the integral in Eq. (3)

$$\int_0^\infty dl a^l \exp[-D(l - l_0)^2] \simeq \exp(l_0 \ln a + \frac{\ln^2 a}{4D})\frac{\sqrt{\pi}}{\sqrt{4D}}[1 + \Phi(\sqrt{D}l_0)] \tag{4}$$
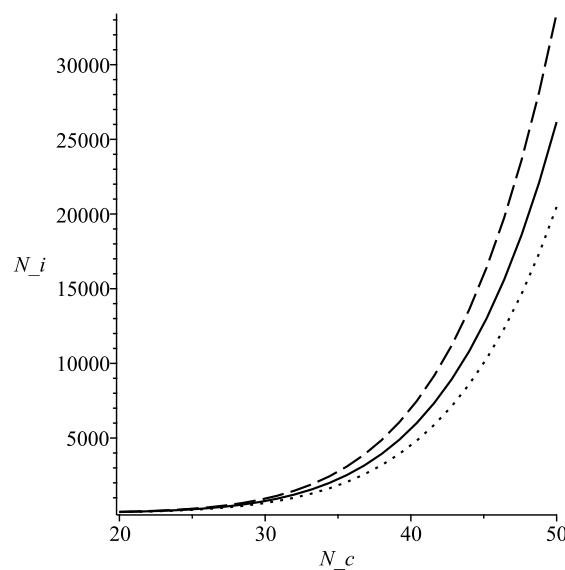
Figure 2: Averaged over l infected cases $\langle N_i(N_c; k; l)\rangle_l$ versus the dangerous contacts N_c for the coefficient of infection transit $k = 0.08$ and the parameter $D = 0.1$ (solid line) and $D = 0.05$ (dashed) in comparison with the curve $N_i(N_c; k = 0.08; l_0 = 7)$ (the dotted line which coincides with the same on Fig. 1)

Therefore for the averaged over the Gauss distribution l number of infected people in population is given by

$$\langle N_i(N_c; k; l)\rangle_l \simeq \exp\left(\frac{\ln^2(N_c \cdot k)}{4D}\right) N_i(N_c; k; l_0), \tag{5}$$

that is generalization of Eq. (1). For $D \to \infty$ we arrive to the result of Eq (1). As is clear the distribution on l always leads to increase of the infected cases $\langle N_i(N_c; k; l)\rangle_l$ in comparison with $N_i(N_c; k; l_0)$. The curves in Figure 2 illustrate the role of averaging over l with different values of the averaging parameter D=0.1 and D = 0.05 (shown, correspondingly, in solid and dashed lines) and their comparison with the non-averaged curve (dotted line - the function $N_i(N_c; k; l_0 = 7)$ with $N_c = 20 \div 50$ and $k = 0.08$. The difference between $\langle N_i(N_c; k; l)\rangle_l$ and $N_i(N_c; k; l_0 = 7)$ increases with N_c increase and can be essential for $N_c > 40$ for the considered parameters.

III.   CONCLUSIONS

The typical for the virus COVID-19 long maximal latent period $2l$ leads to the fast exponential increase of the average infected people with the exponent l. The subsequent restriction of the infection spread is related with the fast and true quarantine actions, which transfer first the exponential increase to saturation, as it is in China at the moment and to further decrease.

Acknowledgment

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