

Forecasting the CoViD–19 Diffusion in Italy and the Related Occupancy of Intensive Care Units

Livio Fenga

*Italian National Institute of Statistics
ISTAT, Rome, Italy 00184
livio.fenga@istat.it*

Abstract:

This paper provides a model-based method for the forecast of the total number of currently CoVoD-19 positive individuals and of the occupancy of the available Intensive Care Units in Italy. The predictions obtained – for a time horizon of 10 days starting from March 29th – will be provided at a national as well as at a more disaggregate levels, following a criterion based on the magnitude of the phenomenon. While the Regions which have been hit the most by the pandemic have been kept separated, the less affected ones have been aggregated into homogeneous macro-areas. Results show that – within the forecast period considered (March 29th - April 7th) – all of the Italian regions will show a decreasing number of CoViD-19 positive people. Same for the number of people who will need to be hospitalized in a Intensive Care Unit (ICU). These estimates are valid under constancy of the Government’s current containment policies. In this scenario, Northern Regions will remain the most affected ones and no significant outbreak are foreseen in the southern regions.

KEYWORDS: Autoregressive Moving Average Models; CoViD–19; Kalman filter, intensive care units, maximum entropy bootstrap.

1. Introduction

On March 19th, the death toll paid by Italy for the spread of the virus CoViD-19 amounted to 3405 deaths, the highest paid by a single country in the World. Despite an hard and relatively timely lock-down policy implemented by the Government, on March 26 this figure has risen to 8165 deaths.

In such an emergency situation, a reliable forecast method for the infection’s development is essential for policy and decision makers to design evidence-based policies and to implement fast actions to curb the spread of the infection. In particular, predicting the number of people currently tested positive for CoViD–19 (thereafter “positive cases”) could be useful to draw the epidemiological curve of the infection and therefore to predict its peak. Other than for this variable, the forecasting procedure presented in this paper is used to predict the future values of another crucial variable, i.e. the number of people needing hospitalization in a Intensive Care Unit (ICU). The Italian ICUs system is at the moment severely stressed due to the spread of the disease, therefore predictions of future ICUs demand could be fruitfully considered in the design and the implementation of operational schemes. The forecast horizon for both the variables is of 10-day starting from March 29th.

Since the Italian regions are affected in different extents by the CoViD-19, it has been decided to perform the forecasting exercise for the following geographical areas: Lombardia, Piedmont, Valle d'Aosta, Veneto, Friuli Venezia Giulia, Trentino Alto Adige, Lazio and Campania. The remaining Regions have been grouped in the following macro-areas: "Center" (Marche, Umbria and Toscana) and "South" (Abruzzo, Molise, Puglia Basilicata, Calabria, Sicilia and Sardegna). At least other two reasons justify such a break down:

1. the different starting times recorded for the lock-downs;
2. the Southern regions have been hit less severely and therefore, especially at the beginning of the observation period, show several zeroes or low numbers across the considered time span.

In essence, in this study the available official data have been employed in a three step procedure, i.e.:

1. data pre-processing, in which data anomalies are identified and corrected according to an approach of the type a Kalman filter;
2. univariate forecasting, based on a autoregressive moving average (ARMA) model for number of positive cases and ICU;
3. bootstrap-based generation of predicted values and confidence intervals.

2. The Data

This paper employs the data related to COVID-19, collected and regularly updated by the Italian National Institute of Health (an agency of the Italian Ministry of Health) and by the Italian Civil Protection Department. The whole data set is freely and publicly available in a comprehensive database, accessible on the Internet at the web address <https://www.iss.it/>. It collects crucial data related to all the persons tested for COVID-19 – from the outbreak of the pandemics (February the 24th) – and, in particular, it

1. is a collection of 21 data points – representing 19 Italian Regions plus the two autonomous provinces of Trento and Bolzano – for each day of infections;
2. considers crucial variables, such as positive cases, recovered cases, deaths, number of people hospitalized and number of people admitted to Intensive Cure Units (ICU).

As already pointed out, in the present study, the variables of interest are the number of people who have been:

1. tested positive for SARS-CoV-2 (in what follows denoted by the bold Latin letter **V**);
2. hospitalized in a ICU (which will be denoted by the bold Latin letter **U**).

It is worth to outlining how, according to the regulations issued by the Italian government, only the people showing moderate to severe symptoms, generally associated with the infection, or which have been in close proximity with at least one positive person, are tested. Therefore, the predictions obtained are to be referred to the sample, as no attempt have been made to carry out inferences procedures for variable estimation at the population level.

In order to correctly process the data, all the regions showing no positive cases at the beginning of the recording period and/or low values along the whole time span have been aggregated into macro areas. This has been done to *i*) give more meaningful results and *ii*) save degrees of freedom (which are always precious in short time series).

In details, the prediction exercise will be performed on the following Regions/macro areas:

A) Northern Regions

1. Lombardia
2. Piedmont
3. Valle d Aosta
4. Veneto
5. Friuli Venezia Giulia
6. Emilia Romagna
7. Liguria
8. Macro Area “Trentino Alto Adige” (Trento and Bolzano)

B) Center Regions

1. Lazio
2. Macro-area “Center”: Marche, Umbria and Toscana

C) Southern Regions

1. Campania
2. Macro-area “South” Abruzzo, Molise, Puglia, Basilicata, Calabria, Sicilia, Sardegna.

The north Italy Regions – presently the more severely affected by the pandemic – have been treated separately along with two other regions, i.e. Lazio and Campania, since their major cities – Rome and Naples – deserve special attention for the institutional role played and the population density exhibited. On the other hand, the Regions showing less worrying figures have been aggregated into macro areas according to their geolocation. The only exception is Valle d’Aosta, which has been left separated as no aggregation options could be found.

To simplify notation, for both the variables of interest V and U , the following convention is introduced:

$${}^K V_j.$$

Here, the upper left superscript (denoted by the upper case Latin letter K) refers to the geographical areas (i.e. North, Center and South) whereas the subscript j is associated to the number the different regions or macroareas are codified with, as above detailed. For example, by the symbols ${}^A V_6$ and ${}^B V_2$ the number of positive cases for the Emilia Romagna region and the Center macro-area “Marche – Umbria – Toscana” are respectively identified.

3. Data Preprocessing

Missing data and other anomalies become the first challenge when designing predictive models, as statistical methods, in general, are designed and tested under the assumption of no missing observations (Moritz et al. (2015)).

Before delving into the details of the proposed procedure, a word of caution is needed since, unfortunately, a visual inspection of the database suggests the presence of a number of anomalous data both at a regional and country level. The detected anomalies might be associated to the biological samples collecting process and the testing procedures. In fact, the typical lab-workflow is governed by a set of rigid protocols which might be critically affected by factors such as the availability of manpower, swabs, reagents and other laboratory materials. In emergency situations, such a workflow can be disrupted and temporal inconsistency might appear as a result. For example, a set of samples might be delivered to a laboratory with longer than usual delays with respect to the time of collection or a given lab can only complete a screening process for a certain number of samples. In both the cases, one day (or more) shift in the release of the lab results can be reasonably expected. A further source of anomalies is represented by the data entry and data editing processes, carried out in working environment necessarily affected by the risk of contagious and under rigid deadlines.

An example of such anomalous data is given in Figure 1, where the series ${}^1V_{1,t}$ (Lombardia) is depicted. Here, some data points showing values inconsistent with the overall pattern are clearly noticeable. Given the (very small) available sample size, the relative weight of such data is almost surely not negligible and can introduce severe distortions in the model parameter inference procedures and thus in the predicted values.

In order to correct those data, a Kalman-smoother state-space model (Simon (2001)) has been applied. In particular, the Kalman smoother adopted is of the type *fixed point smoothing*. This algorithm is designed to obtain the estimate of a realization \hat{w}_N (the time t_N is fixed $N < K$) of a given random variable W_t , given a set of observations $Z_k = \{z_k | 0 \leq N \leq k\}$ Sage and Melsa (1971).

In figure 2 the corrected version of the series ${}^1V_{1,t}$ – resulting by applying the Kalman smoother – is depicted. Not only this series lends itself to a better visual inspection but, more importantly, is more suitable to be processed by the adopted prediction model.

4. Theoretical framework

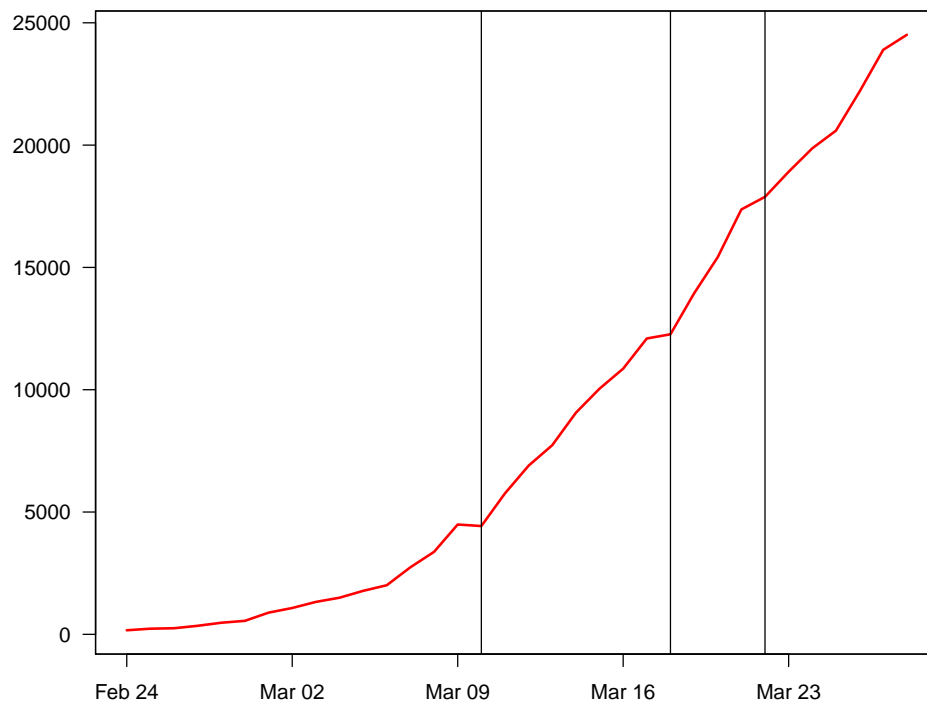
The approach used in this paper relies on *i*) the theory of stochastic process and *ii*) a resampling method. While the former is necessary to generate the input (predicted values) of the bootstrap algorithm, as well as to justify the employment of the outlier correction method, the latter serves the purpose of

1. generating the final predictions, which are affected by a reduced amount of uncertainty (with respect to those generated by the stochastic model)
2. yielding the related confidence intervals.

4.1. The stochastic processes paradigm

The approach proposed in the present paper relies on the assumption that the (transformed) time series ${}^K V_j, t$ and ${}^K V_j, t$ are approximately a realization of a process of the type ARMA (Autoregressive Moving Average) (Makridakis and Hibon (1997)).

Fig. 1. Number of people tested positive (Lombardia): original data



Let $X = (X_t)_{t \in \mathbb{Z}}$ be a real 2^{nd} order stationary process, it is said to admit a ARMA(p,q) representation ($p, q \in \mathbb{Z}$) if, for some constant $a_1, \dots, a_p, b_1, \dots, b_q$, will be:

$$\sum_{j=0}^p a_j X(t-j) = \sum_{j=0}^q b_j \varepsilon(t-j) \quad (t \in \mathbb{Z}), \quad a_0 = b_0 = 1 \quad (1)$$

under the following conditions:

$$E\{\varepsilon(t)|F_{t-1}\} = 0 \quad , \quad E\{\varepsilon^2(t)|F_{t-1}\} = \sigma^2$$

$$E\varepsilon^4(t) < \infty$$

$$\sum_{j=0}^p a_j Z^j \neq 0 \quad , \quad \sum_{j=0}^q b_j Z^j \neq 0 \quad , \quad |Z| \leq 1$$

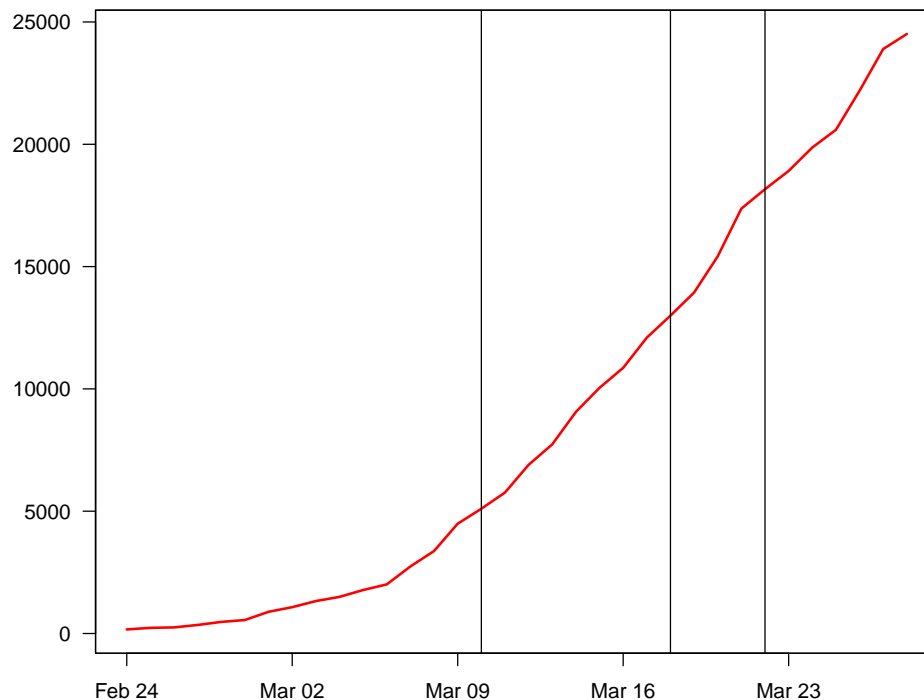
Here F_t denotes the sigma algebra induced by $\varepsilon(j)$, $j \leq t$ and $\sum_{j=0}^p a_j Z^j$ and $\sum_{j=0}^q b_j Z^j$ are assumed not to have common zero.

The above conditions assure that X_t can be represented as:

$$X(t) = \sum_{j=0}^{\infty} \beta_j \varepsilon(t-j) \quad , \quad \sum_{j=0}^{\infty} \beta_j Z^j = (\sum_{j=0}^p a_j Z^j)^{-1} \sum_{j=0}^q b_j Z^j$$

with β_j decreasing to 0 at geometric rate.

Fig. 2. Number of people tested positive (Lombardia): data adjusted via Kalman filter



The dynamics of the series under investigations are not suitable for this theoretical framework which requires 2^{nd} - order stationarity; this is achieved by pre-processing the series according to the following filter: $\log(\nabla^d)$, being the symbol ∇ the difference operator and the exponent d indicating the order of the difference. To fully understand the role played by ∇ , the backward operator B is now introduced. In essence, B moves the time index of an observation back by p time intervals, i.e. $B^p x_t = X_{t-p}$, and thus we have that

$$\nabla^d \log(X_t) = (1 - B)^d \log(X_t). \quad (2)$$

4.2. The Resampling Method

In order to extract valuable information from our data and, at the same time, decrease the total amount of uncertainty associated to the outcomes of the ARMA model, a resampling procedure has been employed. Among the several resampling methods for dependent data available – many of which freely and publicly available in the form of powerful routines working under software packages such as Python® or R® – the adopted resampling method is of the type Maximum Entropy Bootstrap (*MEB*). Proposed by Vinod (2006) and subsequently improved (see, e.g., Vinod (2016)), it is based on basic assumptions which are different from those usually followed by standard schemes. In more details, while in the classic bootstrap an ensemble Ω represents the population of reference the observed time series is drawn from, in *MEB* a large number of ensembles (subsets), say $\{\omega_1, \dots, \omega_N\}$ becomes the elements belonging to Ω , each of them containing a large number of replicates $\{x_1, \dots, x_J\}$.

Unlike standard bootstrap schemes, in the *MEB* case the resample set Ω mimics the observed realization of the underlying stochastic process, in *MEB* a large number of subsets, say $\{\omega_1, \dots, \omega_N\}$ becomes the elements belonging to Ω , each of them containing a large number of replicates $\{x_1, \dots, x_J\}$. Among the important features of the *MEB* scheme, it is worth mentioning the consistency of its bootstrap samples with the ergodic theorem (see, e.g., [Birkhoff \(1931\)](#)) and with the probabilistic structure of the observed time series. In Figure 3 an example of the application of *MEB* for the variable ${}^1V_{t,1}$ is given.

5. The forecasting method

In what follows, the proposed procedure is presented in a step-by-step fashion.

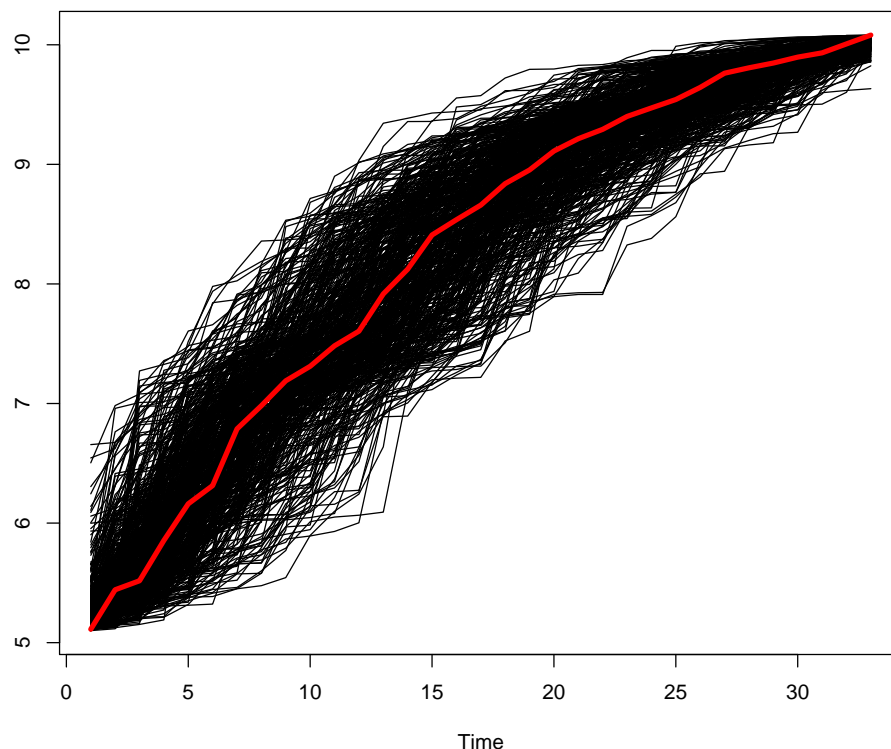
1. Eqn. 1 is estimated for both V_t and U_t so that the model orders (Eqn. 1) M_1 and M_2 become available;
2. for each time series V_t and U_t the *MEB* procedure is applied so that the sets \mathcal{V} and \mathcal{U} – each containing $B = 500$ “bona fide” replications – are available, i.e. $\mathcal{V} \equiv [V_1^*, V_2^*, \dots, V_B^*]$ and $\mathcal{U} \equiv [U_1^*, U_2^*, \dots, U_B^*]$ (in Figure 4 the set \mathcal{V} for the variable ${}^1V_{t,1}$ is given);
3. for each of the replications stored in \mathcal{V} , Eqn. 1 is estimated according to the model order selected, i.e. M_1 , and the 1 to 10–step–ahead predictions – as well as the 5% and 95% bootstrap confidence interval – are generated;
4. the B predictions and the confidence intervals obtained in the previous step are stored in the $B \times 3$ matrix $[F_{\mathcal{V}}(h); h = 1, 2, \dots, 10]$, whose columns are: lower bootstrap confidence interval, bootstrap prediction and upper bootstrap confidence interval, respectively denoted by the symbols $CI_{L,b}^*(h)$, $V_{t,b}^*$, $CI_{U,b}^*(h)$ $b = 1, \dots, B$.
5. the median value $\hat{V}^* = \mathbb{M}(V_{t,b}^*)$ is then extracted along with the $\approx 95\%$ confidence intervals $CI_{L,b}^*(h = 1)$ and $CI_{U,b}^*(h = 1)$, computed according to the t–percentile method. The explanation of this procedure goes beyond the scope of this paper, therefore the interested reader is referred to the excellent paper by [Berkowitz and Kilian \(2000\)](#).
6. $CI_L^*(h = 2, \dots, 10)$ and $CI_U^*(h = 2, \dots, 10)$ (the subscript b is omitted for brevity) are computed conditional to a subset of \mathcal{V} , say $\tilde{\mathcal{V}}$, made up of the bootstrap replications whose range falls between the minimum and maximum values of the values of the confidence intervals computed for $h = 1$. In symbols:

$$\min(CI_U^*(h = 1)) \leq [\tilde{\mathcal{V}} \subset \mathcal{V}] \leq \max(CI_U^*(h = 1)); \quad (3)$$

7. steps 1–6 are repeated for U_t , so that a new matrix of prediction of dimension $B \times 3$ is built, i.e. $[F_{\mathcal{U}}(h) h = 1, 2, \dots, 10]$, whose columns are as in $F_{\mathcal{V}}(h)$ and denoted by the symbols $CI_U^*(h)$, \hat{U}_t^* , $CI_U^*(h)$.

Unfortunately, the whole procedure cannot be considered fully automatic since the estimation of Eqn. 1 (step 1) is required.

Fig. 3. Lombardia: B= 500 Bootstrap replications performed via the MEB algorithm on the adjusted, log-transformed, data (in red the original time series is depicted)



5.1. The adopted models

The stochastic model structures identified for both V_t and U_t are almost always of the type ARMA (1,0), with the exception of Campania (ARMA(0,1), for both the variables V_t and U_t) and Emilia Romagna, for which the best model for the variable U_t is of the type ARMA(1,1). The most suitable prefilter (Eqn. 2) has been always of the type $d=3$ difference of the natural log of the variables of interest.

6. Empirical evidences

At the national level (data have been plotted in Figure 4) the peak in the number of CoViD-19-positives will be reached on 2 April, with a number of predicted positive close to 77,000. The maximum forecasted value for the occupied ICU – expected for April 4 – will be 4280. These values have been calculated using an indirect methodology, i.e. by summing up the estimates obtained at a disaggregated level. Regarding the results obtained at a disaggregated level, the models outcomes are now commented.

- Lombardia – the most affected region – will reach the peak of positive cases (25963) and of the demand of ICUs (1425) respectively on 2 and 4 April;
- Emilia Romagna is the second most affected Region by COVID-19 but still shows a very high number of victims. The trend of infected people will reach its peak on April 5th whereas the number of cases in Intensive Care will continue to grow at a progressively slower rate over the forecasting period;

- Veneto is the third Region for number of deaths. Here, the number of positive cases, as well as the number of cases in ICU, will reach the peak on April 3rd;
- for Piedmont – the fourth Region for number of victims – the predicted positive cases will reach the peak on March 29th (6635) whereas the persons in ICU will be 431 on 31 March, when the peak is predicted;
- Liguria will begin a process of relative reduction of positive cases as early as March 29th. The number of cases in Intensive Care, after a period of stability (lasting until 31 March), will start a slow decreasing path;
- Positive cases in Trentino Alto Adige – which incorporates the cities of Trento and Bolzano – are projected to be 2158 on March 30th and then a decreasing trend is expected. The ICU beds occupied in this region will reach its peak on around 3 April;
- The positive cases in Friuli Venezia Giulia show a relatively stable trend in the first half of the prediction interval with a peak around April 4th, after that the absolute number of cases will start decreasing. The number of cases in ICU will reach the peak between 30 March and 1 April;
- Valle D’Aosta is a small region which has been relatively less impacted by the virus. Here, a downward trend is expected to start on March 31st (for the positive cases) and around 31 March (cases in ICUs);
- The upward trend in the number of positive cases of Lazio is estimated to stop on 31 March and to reach the minimum at the end of the forecasting period (1821 cases). The number of ICU cases is estimated to reach its peak on the period 1–3 April;
- The Macro-area Center will reach its peak at the very beginning of the month of April (for the variable V) whereas for the variable U the estimated peak day is around 31 March;
- Campania will reach the peak of contagions on April 5 whereas ICU cases will do on the previous day;
- The remaining southern regions (Abruzzo, Molise, Puglia, Basilicata, Calabria, Sicily and Sardinia) will show an upward trend in the number of future positive cases lasting until 6 April, where 6355 cases are predicted. The number of persons requiring an ICU will reach the peak on 4 April (348 is the estimated number of cases).

Fig. 4. Italy, time series data of positive (data corrected via Kalman filter, left side axis) and of the number of people hospitalized in ICUs (right side axis)

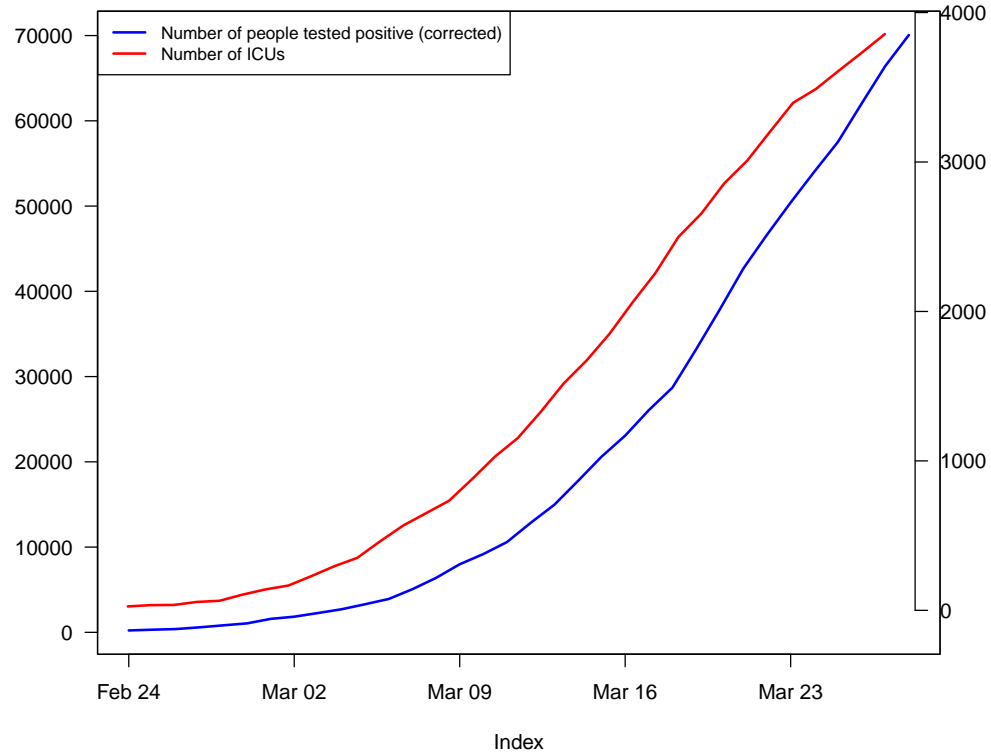


Table 1. 10-step-ahead predictions for the variable V_t , number of persons tested positive

	Italy		
	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	66504	71586	76647
30 March	69753	73409	79373
31 March	70760	75696	82773
1 April	70817	76723	86430
2 April	69925	77811	90800
3 April	69309	76952	96806
4 April	68480	75927	102946
5 April	66531	74860	109265
6 April	64940	75006	117550
7 April	60197	74875	126205
	Lombardia		
	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	22538	25214	26384
30 March	23296	25456	26790
31 March	23969	25792	27565
1 April	23864	25842	27634
2 April	23662	25963	28230
3 April	23209	25675	28342
4 April	22709	25717	28781
5 April	21964	25067	29524
6 April	20802	24431	30131
7 April	19619	23752	30628
	Piedmont		
	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	6115	6635	7017
30 March	6264	6610	7193
31 March	6209	6568	7481
1 April	6079	6401	7942
2 April	5837	6152	8271
3 April	5502	5898	8590
4 April	5134	5566	8753
5 April	4666	5135	8995
6 April	4178	4732	9279
7 April	3707	4289	9417

Liguria

	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	2026	2062	2217
30 March	2007	2048	2242
31 March	1955	2019	2361
1 April	1891	1969	2416
2 April	1804	1900	2515
3 April	1701	1805	2580
4 April	1589	1709	2645
5 April	1470	1590	2748
6 April	1344	1440	2917
7 April	1214	1311	3090

Valle D'Aosta

	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	404	480	512
30 March	410	491	524
31 March	417	501	546
1 April	420	500	551
2 April	411	494	551
3 April	382	483	542
4 April	359	462	526
5 April	339	443	541
6 April	301	412	530
7 April	267	383	516

Veneto

	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	6346	7145	7559
30 March	6569	7213	7749
31 March	6706	7431	8067
1 April	6721	7463	8144
2 April	6535	7539	8415
3 April	6414	7469	8758
4 April	6073	7374	9168
5 April	5703	7182	9214
6 April	5303	6842	9593
7 April	4798	6537	10203

Friuli Venezia Giulia Infetti

	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	1029	1124	1167
30 March	1083	1138	1195
31 March	1100	1169	1226
1 April	1104	1189	1269
2 April	1109	1212	1267
3 April	1097	1209	1272
4 April	1060	1230	1287
5 April	1035	1203	1305
6 April	1005	1202	1333
7 April	976	1208	1343

Emilia Romagna

	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	9163	10500	11265
30 March	9464	10975	11717
31 March	9535	11360	12176
1 April	9587	11613	12562
2 April	9476	11815	12954
3 April	9276	11943	13291
4 April	9002	12132	13539
5 April	8528	12244	13720
6 April	8034	12125	13995
7 April	7311	11966	14189.9250136506

Trentino Alto Adige

	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	2021	2147	2324
30 March	2076	2158	2488
31 March	2114	2124	2617
1 April	2110	2083	2707
2 April	2097	1988	2787
3 April	2064	1868	2764
4 April	2023	1725	2742
5 April	1944	1548	2751
6 April	1851	1373	2718
7 April	1707	1200	2641

Lazio

	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	2013	2208	2355
30 March	2160	2244	2436
31 March	2182	2298	2553
1 April	2153	2279	2666
2 April	2161	2256	2785
3 April	2140	2222	2991
4 April	2100	2154	3132
5 April	2041	2046	3233
6 April	1908	1956	3485
7 April	1768	1821	3782

Macro Area Center

	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	6822	7499	7911
30 March	7104	7671	8087
31 March	7130	7864	8396
1 April	7001	7896	8741
2 April	6758	7889	8987
3 April	6424	7851	9156
4 April	6078	7813	9695
5 April	5512	7718	10081
6 April	4925	7494	10580
7 April	4397	7238	10941

Campania

	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	1169	1453	1549
30 March	1267	1494	1662
31 March	1293	1527	1747
1 April	1287	1548	1835
2 April	1280	1582	1898
3 April	1252	1619	1953
4 April	1222	1640	1972
5 April	1160	1649	2020
6 April	1090	1648	2063
7 April	994	1645	2145

Macro Area Sud

	$CI_L^*(h)$	\hat{V}_t^*	$CI_U^*(h)$
29 March	4380	5226	5662
30 March	4646	5406	5846
31 March	4689	5648	6246
1 April	4705	5791	6528
2 April	4496	5931	6902
3 April	4122	6129	7325
4 April	3841	6169	7695
5 April	3445	6346	8141
6 April	2981	6355	8787
7 April	2512	6345	9397

Table 2. 10-step-ahead predictions for the variable U_t (people in ICUs)

Italy	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	3681	3960	4086
30 March	3751	4009	4170
31 March	3730	4075	4264
1 April	3732	4141	4350
2 April	3707	4185	4386
3 April	3667	4262	4493
4 April	3608	4280	4610
5 April	3492	4275	4701
6 April	3347	4249	4814
7 April	3143	4243	4899

Lombardia	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	1272	1369	1426
30 March	1286	1389	1458
31 March	1287	1402	1482
1 April	1291	1412	1490
2 April	1289	1420	1509
3 April	1286	1427	1526
4 April	1270	1425	1537
5 April	1252	1421	1558
6 April	1227	1416	1578
7 April	1191	1412	1599

Piedmont	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	422	433	451
30 March	428	436	457
31 March	428	438	470
1 April	427	436	478
2 April	4241	434	491
3 April	419	427	498
4 April	409	417	508
5 April	395	406	512
6 April	373	396	523
7 April	354	383	535

Liguria	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	156	163	168
30 March	159	163	171
31 March	158	163	174
1 April	158	162	178
2 April	158	161	181
3 April	156	159	185
4 April	153	157	187
5 April	148	153	191
6 April	143	151	193
7 April	137	145	195

Valle Aosta

	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	24	25	27
30 March	25	25	29
31 March	24	25	30
1 April	24	24	31
2 April	23	22	32
3 April	22	21	33
4 April	21	19	34
5 April	19	16	36
6 April	18	14	36
7 April	15	12	37

Veneto

	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	331	357	367
30 March	335	360	375
31 March	335	365	382
1 April	334	370	391
2 April	332	375	401
3 April	324	378	409
4 April	312	375	424
5 April	298	376	440
6 April	286	370	448
7 April	267	369	461

Friuli Venezia Giulia

	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	56	57	60
30 March	56	58	60
31 March	56	58	63
1 April	55	58	63
2 April	54	57	65
3 April	52	56	66
4 April	51	56	67
5 April	49	55	69
6 April	48	53	71
7 April	44	52	72

Emilia Romagna

	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	306	349	379
30 March	309	365	394
31 March	309	369	409
1 April	312	372	421
2 April	313	379	433
3 April	314	386	439
4 April	311	388	442
5 April	307	392	442
6 April	298	399	447
7 April	288	401	444

Trentino Alto Adige

	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	110	124	135
30 March	113	127	144
31 March	114	130	150
1 April	113	132	155
2 April	113	133	158
3 April	111	133	160
4 April	107	131	161
5 April	101	130	164
6 April	94	126	171
7 April	87	123	175

Lazio

	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	120	136	146
30 March	125	139	157
31 March	123	141	171
1 April	118	144	179
2 April	111	145	200
3 April	104	145	219
4 April	95	141	235
5 April	84	139	264
6 April	73	134	294
7 April	61	131	343

Macro Area Center

	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	476	491	509
30 March	485	492	519
31 March	484	492	534
1 April	482	489	547
2 April	479	484	565
3 April	475	478	586
4 April	468	469	608
5 April	460	460	624
6 April	448	448	649
7 April	438	438	681

Campania

	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	126	142	167
30 March	133	159	197
31 March	138	176	228
1 April	144	189	257
2 April	147	201	293
3 April	148	206	326
4 April	143	211	348
5 April	134	203	382
6 April	120	197	411
7 April	109	192	447

Macro Area Sud

	$CI_L^*(h)$	\hat{U}_t^*	$CI_U^*(h)$
29 March	296	319	342
30 March	303	326	348
31 March	312	335	367
1 April	306	340	367
2 April	302	343	383
3 April	298	346	394
4 April	289	348	408
5 April	278	342	421
6 April	264	334	441
7 April	244	318	462

7. Disclaimer

The views and opinions expressed in this article are those of the author and do not necessarily reflect the official policy or position of the Italian National Institute of Statistics.

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